Network Theory

for

EC / EE / IN

By

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Syllabus for Networks


Analysis of GATE Papers

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- Answer Keys & Explanations  

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Network Solution Methodology

Learning Objectives
After reading this chapter, you will know:
1. Series and Parallel Connection of Circuit Elements
2. Voltage and Current Relation of Network
3. KCL, KVL
4. Mesh Analysis
5. Nodal Analysis
6. Voltage and Current Sources
7. Dependent Sources
8. Power and Energy
9. Network Theorems
10. Delta to Star Transformation

Introduction
The passive circuit elements resistance \( R \), inductance \( L \) and capacitance \( C \) are defined by the manner by which voltages and currents are related for the individual element. The table below summarizes the voltage-current (V-I) relation, instantaneous power (P) consumption and energy stored in the period \([t_1, t_2]\) for each of above elements.

<table>
<thead>
<tr>
<th>SL No.</th>
<th>Circuit Element</th>
<th>Symbol in Electric Circuit</th>
<th>Units</th>
<th>Voltage – Current Relation</th>
<th>Instantaneous Power, P</th>
<th>Energy Stored / Dissipated in ([t_1, t_2])</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>Resistance, ( R )</td>
<td>( \frac{i}{V} )</td>
<td>Ohm (( \Omega ))</td>
<td>( V = i \cdot R ) (Ohm’s law)</td>
<td>( Vi = i^2 R )</td>
<td>( i^2 R(t_2 - t_1) )</td>
</tr>
<tr>
<td>2</td>
<td>Inductance, ( L )</td>
<td>( \frac{i}{V} )</td>
<td>Henry (( H ))</td>
<td>( V = L \frac{di}{dt} )</td>
<td>( Li \frac{di}{dt} )</td>
<td>( \frac{1}{2} L(i_{t_2}^2 - i_{t_1}^2) )</td>
</tr>
<tr>
<td>3</td>
<td>Capacitance, ( C )</td>
<td>( \frac{i}{V} )</td>
<td>Farad (( F ))</td>
<td>( i = C \frac{dv}{dt} )</td>
<td>( Cv \frac{dv}{dt} )</td>
<td>( \frac{1}{2} C(v_{t_2}^2 - v_{t_1}^2) )</td>
</tr>
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</table>

Table 1.1 Voltage – Current Relation of Network Elements

“Success consists of going from failure to failure without loss of enthusiasm.”
...Winston Churchill
In the above table, if \( i_m \) is the current at instant \( m \) and \( V_m \) is the voltage at instant \( m \), total energy dissipated in a resistor (R) in \([ t_1, t_2 ]\) is given by:

\[
\int_{t_1}^{t_2} i^2 R \, dt = \int_{t_1}^{t_2} v^2 \, dt
\]

**Series and Parallel Connection of Circuit Elements**

Figure below summarizes equivalent resistance / inductance / capacitance for different combinations of network elements.

- **Series Connection of Resistor**
  
  \[
  R = R_1 + R_2 + \ldots + R_n
  \]

- **Parallel Connection of Resistors**
  
  \[
  \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}
  \]

**Voltage / Current Relation in Series / Parallel Connection of Resistor**

Figures below summarize voltage/current relations in series and parallel connection of resistors.

- **Series Connection of Resistors**
  
  \[
  i_1 = i_2 = \ldots = i_n = i
  \]

  \[
  V = \sum_{i=1}^{n} V_i
  \]

  \[
  V = \frac{V \times R_i}{\sum_{i=1}^{n} R_i}
  \]

  \[
  V = \frac{V_i}{R_i}
  \]

- **Parallel Connection of Resistors**
  
  \[
  \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n}
  \]

  \[
  R = \frac{R_1 R_2 \ldots R_n}{R_1 + R_2 + \ldots + R_n}
  \]

  \[
  V = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \ldots + \frac{V_n}{R_n}
  \]

  \[
  V = \sum_{i=1}^{n} \frac{V_i}{R_i}
  \]
For parallel connection of resistors,

\[ i = \Sigma i_i, i_i = \frac{i}{R_i} \quad \forall \ i = 1, \ldots, n \]

\[ V_1 = V_2 = \ldots = V_n = V \]

**Kirchoff’s Current Law (KCL)**

The algebraic sum of current at a node in a electrical circuit is equal to zero.

At any point in electrical circuit the phasor sum of the current flowing towards a junction is equal to the phasor sum of the currents flowing away from the junction.

**Demonstration of KCL**

Assuming that current approaching node ‘O’ bears positive sign, and vice versa,

\[ i_1 - i_2 + i_3 + i_4 - i_5 = 0 \]

At any point in an electrical circuit the phasor sum of the currents flowing towards that junction is equal to the phasor sum of the current flowing away from the junction.

Node = Junction

**Kirchoff’s Voltage Law (KVL)**

In any closed loop electrical circuit, the algebraic sum of voltage drops across all the circuit elements is equal to emf rise in the same. Figure below demonstrates KVL and KCL equation can be written as,

\[ iR_1 + iR_2 = E \]

In any closed loop in a networks the phasor sum of the voltage drops (i.e., product of current and impedance) taken around the loop is equal to the phasor sum of the emf’s acting in that loop.

**Demonstration of KVL**
From Kirchoff’s current law, \( i_3 = i_1 - i_2 \)
Also from KVL,
\[
\begin{align*}
   i_1 R_1 + (i_1 - i_2)R_2 &= E \\
   i_2 R_3 + i_2 R_4 - (i_1 - i_2)R_2 &= 0
\end{align*}
\]

**Mesh Analysis**

In the mesh analysis, a current is assigned to each window of the network such that the currents complete a closed loop. They are also referred to as loop currents. Each element and branch therefore will have an independent current. When a branch has two of the mesh currents, the actual current is given by their algebraic sum. Once the currents are assigned, Kirchhoff’s voltage law is written for each of the loops to obtain the necessary simultaneous equations.

Use mesh analysis to find \( I_1 \) and \( I_2 \),
\[
\begin{align*}
   i_1 R_1 + (i_1 - i_2)R_2 &= E_1 \Rightarrow i_1 (R_1 + R_2) - i_2 R_2 = E_1 \\
   i_2 R_3 + i_2 R_4 + (i_2 - i_1)R_2 &= 0 \Rightarrow i_2 (R_2 + R_3 + R_4) - i_1 R_2 = 0
\end{align*}
\]
The above set of simultaneous equations should be solved for \( I_1 \) and \( I_2 \).

**Solution of Simultaneous Equations**

**Matrix Inversion Method**

The above set of equations can be written as below in matrix form:
\[
\begin{bmatrix}
   R_1 + R_2 & -R_2 \\
   -R_2 & R_2 + R_3 + R_4
\end{bmatrix}
\begin{bmatrix}
   I_1 \\
   I_2
\end{bmatrix} =
\begin{bmatrix}
   E_1 \\
   0
\end{bmatrix}
\]
This is of form, \( AX = B \Rightarrow X = A^{-1} B \)
Where, \( X = \begin{bmatrix} I_1 & I_2 \end{bmatrix}^T \)

**Cramer’s Rule**

To find either of \( I_1 \) and \( I_2 \), use Cramer’s rule as below,
\[
I_1 = \frac{\begin{bmatrix}
   E_1 & -R_2 \\
   0 & R_2 + R_3 + R_4
\end{bmatrix}}{|A|}
\]
Mesh Analysis (Using Super Mesh)
When two of the loops have a common element as a current source, mesh analysis is not applied to both loops separately. Instead both the loops are merged and a super mesh is formed. Now KVL is applied to super mesh. For the circuit in figure shown below,
\[ I_1 R_1 + I_2 R_2 = E_1 \]
\[ I_2 - I_1 = 1 \]

Nodal Analysis
Typically, electrical networks contain several nodes, where some are simple nodes and some are principal nodes. In the node voltage method, one of the principal nodes is selected as the reference and equations based on KCL are written at the other principal nodes. At each of these other principal nodes, a voltage is assigned, where it is understood that this voltage is with respect to the reference node. These voltages are the unknowns and are determined by Nodal Analysis.

Use nodal analysis to find \( V_1 \),
\[ \frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - E_2}{R_3} = 0 \]

When the node voltages to be found by nodal analysis are more than 1, the node voltages can be found from simultaneous equations by matrix inversion method or Cramer’s rule.

Nodal Analysis (Including Super Node)
When ideal voltage source is connected between two non-reference Node, then it is easy to get solution using Super Node Technique. i.e., instead of solving Nodal equations separately they were merged and treated as a single Node.
By nodal analysis,
\[
\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4} = 0
\]
\[
V_2 - V_1 = E
\]

**Voltage and Current Source**

**Ideal vs Practical Voltage Source**

Figure above depicts the symbol of a practical voltage source. Here E is the emf of source and \( R_1 \) is the internal resistance of the source. For an ideal voltage source, \( R_1 \) is zero and for a practical source, \( R_1 \) is finite & small.

**Ideal vs Practical Current Source**

Figure above depicts the symbol of a practical current source. Here, I is the current of source and \( R_1 \) is internal resistance of source. For an ideal voltage source, \( R_1 \) is infinite and for a practical source, \( R_1 \) is finite and large.

**Dependent Sources**

A source is called dependent if voltage / current of the source depends on voltage / current in some other part of the network. Depending upon the nature of the source, dependent sources can be classified as below.

**Voltage Controlled Voltage Source (VCVS)**

Here the voltage of the voltage source depends on voltage across some other element in the network.