Network Theory

for

EC / EE / IN

By

THE GATE ACADEMY

A Forum of IIT / IISc Graduates

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Syllabus for Networks

Network Graphs: Matrices Associated With Graphs: Incidence, Fundamental Cut Set and Fundamental Circuit Matrices. Solution Methods: Nodal and Mesh Analysis. Network Theorems: Superposition, Thevenin and Norton's, Maximum Power Transfer, Wye-Delta Transformation. Steady State Sinusoidal Analysis Using Phasors. Linear Constant Coefficient Differential Equations, Time Domain Analysis of Simple RLC Circuits, Solution of Network Equations Using Laplace Transform, Frequency Domain Analysis of RLC Circuits. 2-Port Network Parameters, Driving Point And Transfer Functions, State Equations For Networks.

Analysis of GATE Papers

Year	ECE	EE	IN
2015	9.33	9.00	11.00
2014	10.00	6.7	10.00
2013	15.00	11.00	13.00
2012	13.00	13.00	16.00
2011	9.00	9.00	6.00
2010	8.00	10.00	6.00
2009	13.00	12.00	2.00
2008	16.00	20.00	18.00
2007	10.00	14.00	14.00
2006	6.00	14.00	9.00
Over All Percentage	10.93%	11.87%	10.50%

Contents

"Success consists of going from failure to failure without loss of enthusiasm."
.... Winston Churchill

CHAPTER 1 Network Solution Methodology

Learning Objectives

After reading this chapter, you will know:

1. Series and Parallel Connection of Circuit Elements
2. Voltage and Current Relation of Network
3. KCL, KVL
4. Mesh Analysis
5. Nodal Analysis
6. Voltage and Current Sources
7. Dependent Sources
8. Power and Energy
9. Network Theorems
10. Delta to Star Transformation

Introduction

The passive circuit elements resistance R, inductance L and capacitance C are defined by the manner by which voltages and currents are related for the individual element. The table below summarizes the voltage-current(V-I) relation, instantaneous power (P) consumption and energy stored in the period $[t_1,t_2]$ for each of above elements.

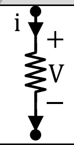
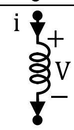
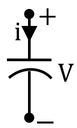
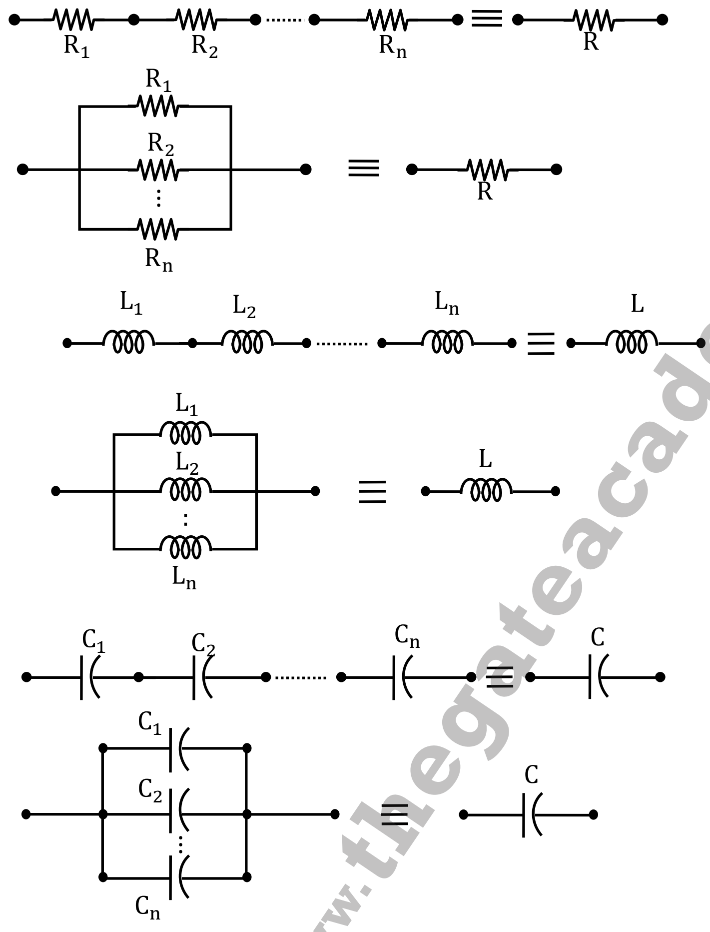
SL No.	Circuit Element	Symbol in Electric Circuit	Units	Voltage – Current Relation	Instantaneous Power , P	Energy Stored / Dissipated in $[t_1, t_2]$
1	Resistance, R		Ohm (Ω)	V= i R (Ohm's law)	Vi= i^2R	$i^2R(t_2 - t_1)$
2	Inductance, L		Henry (H)	$V = L\frac{di}{dt}$	$Li\frac{di}{dt}$	$\frac{1}{2}L(i_2^2 - i_1^2)$
3	Capacitance, C		Farad (F)	$i = C\frac{dv}{dt}$	$Cv\frac{dv}{dt}$	$\frac{1}{2}C(v_2^2 - v_1^2)$

Table 1.1 Voltage –Current Relation of Network Elements

In the above table, if i_m is the current at instant m and V_m is the voltage at instant m, total energy dissipated in a resistor (R) in $[t_1, t_2] = \int_{t_1}^{t_2} v_t\, i_t\, dt = \int_{t_1}^{t_2} i_t^{\,2}\, R\, dt$

Series and Parallel Connection of Circuit Elements

Figure below summarizes equivalent resistance /inductance /capacitance for different combinations of network elements.



$$R = R_1 + R_2 + \dots\dots\dots\dots. + R_n$$

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \dots\dots\dots\dots. + \frac{1}{R_n}$$

$$L = L_1 + L_2 + \dots\dots\dots\dots. + L_n$$

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \dots\dots\dots\dots. + \frac{1}{L_n}$$

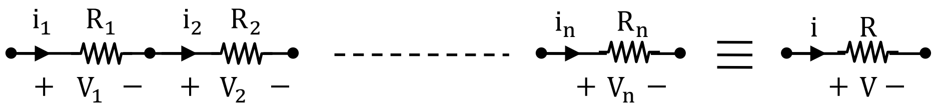
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \dots\dots\dots\dots. + \frac{1}{C_n}$$

$$C = C_1 + C_2 + \dots\dots\dots\dots. + C_n$$

Series and Parallel Connection of Circuit Elements

Voltage / Current Relation in Series / Parallel Connection of Resistor

Figures below summarize voltage/current relations in series and parallel connection of resistors.

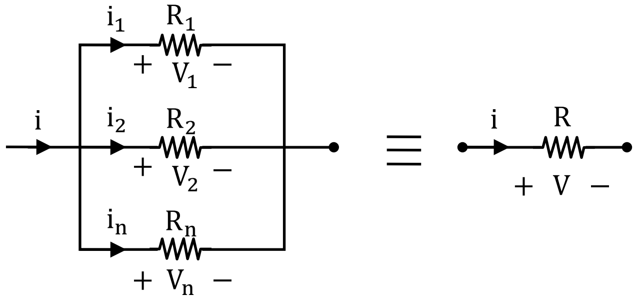


Series Connection of Resistor

For series connection of resistors, $i_1 = i_2 = \dots\dots.\ i_n = i$

$$V_i = \frac{V \times R_i}{\sum_{i=1}^{n} R_i}, \forall i = 1, \dots\dots\dots n$$

$$\left[V = \sum_{i=1}^{n} V_i\right]$$



Parallel Combination of Resistors

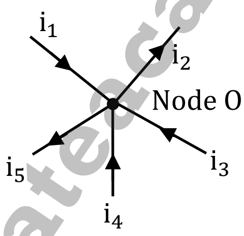
For parallel connection of resistors,

$$i = \Sigma i_i \,, i_i = i \times \frac{\frac{1}{R_i}}{\left(\Sigma \frac{1}{R_i}\right)} \quad \forall\, i = 1, \ldots\ldots\ldots n$$

$$V_1 = V_2 = \ldots\ldots\ldots\ldots\ldots\ldots = V_n = V$$

Kirchoff's Current Law (KCL)

The algebraic sum of current at a node in a electrical circuit is equal to zero.
At any point in electrical circuit the phasor sum of the current flowing towards a junction is equal to the phasor sum of the currents flowing away from the junction.



Demonstration of KCL

Assuming that current approaching node 'O' bears positive sign, and vice versa,

$i_1 - i_2 + i_3 + i_4 - i_5 = 0$

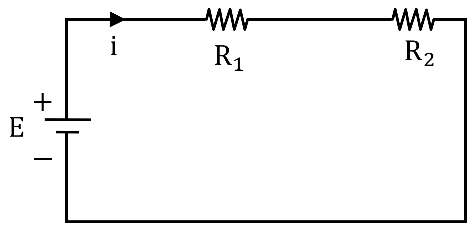
At any point in an electrical circuit the phasor sum of the currents flowing towards that junction is equal to the phasor sum of the current flowing away from the junction.
Node = Junction

Kirchoff's Voltage Law (KVL)

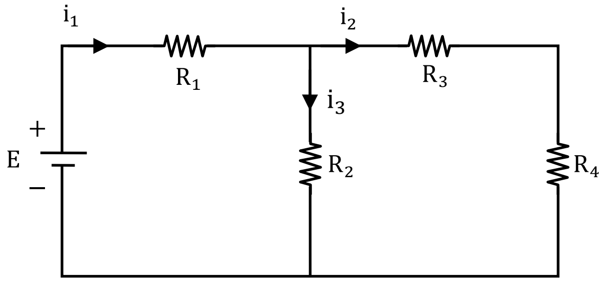
In any closed loop electrical circuit, the algebraic sum of voltage drops across all the circuit elements is equal to emf rise in the same. Figure below demonstrates KVL and KCL equation can be written as,

$iR_1 + iR_2 = E$

In any closed loop in a networks the phasor sum of the voltage chops (i.e., product of current and impedance) taken around the loop is equal to the phasor sum of the emf's acting in that loop.



Demonstration of KVL

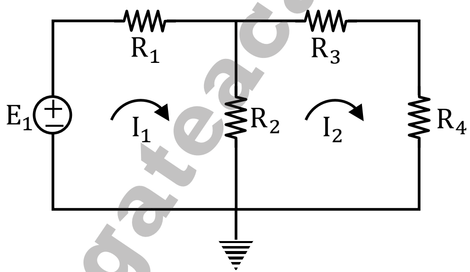


Demonstration of KCL & KVL

From Kirchoff's current law, $i_3 = i_1 - i_2$
Also from KVL,
$i_1R_1 + (i_1 - i_2)R_2 = E$
$i_2R_3 + i_2R_4 - (i_1 - i_2)R_2 = 0$

Mesh Analysis

In the mesh analysis, a current is assigned to each window of the network such that the currents complete a closed loop. They are also referred to as loop currents. Each element and branch therefore will have an independent current. When a branch has two of the mesh currents, the actual current is given by their algebraic sum. Once the currents are assigned, Kirchhoff's voltage law is written for each of the loops to obtain the necessary simultaneous equations.



Demonstration of Mesh Analysis

Use mesh analysis to find I_1 and I_2,
$I_1R_1 + (I_1 - I_2)\,R_2 = E_1 \Rightarrow I_1(R_1 + R_2) - I_2R_2 = E_1$
$I_2R_3 + I_2R_4 + (I_2 - I_1)R_2 = 0 \Rightarrow I_2\,(R_2 + R_3 + R_4) - I_1R_2 = 0$
The above set of simultaneous equations should be solved for I_1 and I_2.

Solution of Simultaneous Equations

Matrix Inversion Method

The above set of equations can be written as below in matrix form:

$$\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}$$

This is of form, $AX = B \Rightarrow X = A^{-1}B$
Where, $X = [I_1 \quad I_2]^T$

Cramer's Rule

To find either of I_1 and I_2, use Cramer's rule as below,

$$I_1 = \frac{\begin{vmatrix} E_1 & -R_2 \\ 0 & R_2 + R_3 + R_4 \end{vmatrix}}{|A|}$$

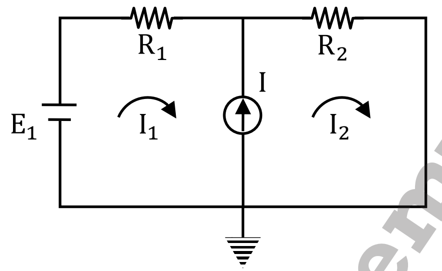
$$I_2 = \frac{\begin{vmatrix} R_1 + R_2 & E_1 \\ -R_2 & 0 \end{vmatrix}}{|A|}$$

Mesh Analysis (Using Super Mesh)

When two of the loops have a common element as a current source, mesh analysis is not applied to both loops separately. Instead both the loops are merged and a super mesh is formed. Now KVL is applied to super mesh. For the circuit in figure shown below,

$I_1R_1 + I_2R_2 = E_1$

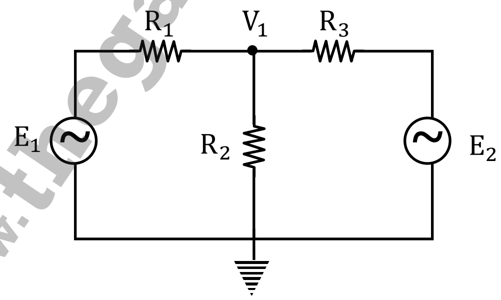
$I_2 - I_1 = I$



Demonstration of Mesh Analysis Using Super Mesh

Nodal Analysis

Typically, electrical networks contain several nodes, where some are simple nodes and some are principal nodes. In the node voltage method, one of the principal nodes is selected as the reference and equations based on KCL are written at the other principal nodes. At each of these other principal nodes, a voltage is assigned, where it is understood that this voltage is with respect to the reference node. These voltages are the unknowns and are determined by Nodal Analysis.



Demonstration of Nodal Analysis

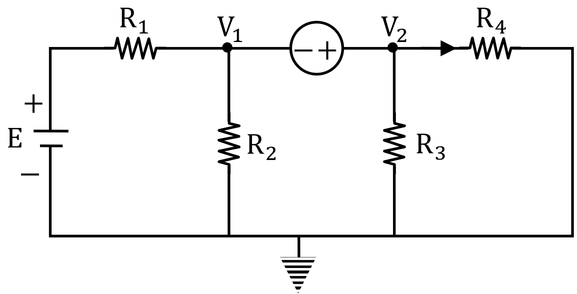
Use nodal analysis to find V_1,

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - E_2}{R_3} = 0$$

When the node voltages to be found by nodal analysis are more than 1, the node voltages can be found from simultaneous equations by matrix inversion method or Cramer's rule.

Nodal Analysis (Including Super Node)

When ideal voltage source is connected between two non-reference Node, then it is easy to get solution using Super Node Technique. i.e., instead of solving Nodal equations separately they were merged and treated as a single Node.



Demonstration of Nodal Analysis Using Super Node

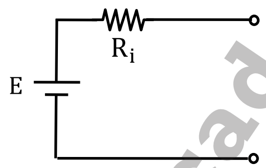
By nodal analysis,

$$\frac{V_1 - E}{R_1} + \frac{V_1}{R_2} + \frac{V_2}{R_3} + \frac{V_2}{R_4} = 0$$

$$V_2 - V_1 = E$$

Voltage and Current Source

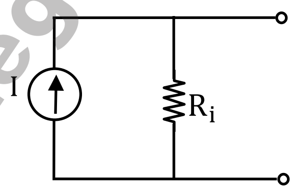
Ideal vs Practical Voltage Source



Practical Voltage Source

Figure above depicts the symbol of a practical voltage source. Here E is the emf of source and R_i is the internal resistance of the source. For an ideal voltage source, R_i is zero and for a practical source, R_i is finite & small.

Ideal vs Practical Current Source



Practical Current Source

Figure above depicts the symbol of a practical current source. Here,I is the current of source and R_i is internal resistance of source. For an ideal voltage source, R_i is infinite and for a practical source, R_i is finite and large.

Dependent Sources

A source is called dependent if voltage / current of the source depends on voltage / current in some other part of the network. Depending upon the nature of the source, dependent sources can be classified as below.

Voltage Controlled Voltage Source (VCVS)

Here the voltage of the voltage source depends on voltage across some other element in the network.