Heat transfer through fins

Introduction

Convection heat transfer between a hot solid surface and the surrounding colder fluid is governed by the Newton’s cooling law which states that “the rate of convection heat transfer is directly proportional to the temperature difference between the hot surface and the surrounding fluid and is also directly proportional to the area of contact or exposure between them”. Newton’s law of cooling can be expressed as

$$Q_{\text{conv}} = h A (T_s - T_\infty)$$

Where, $h$ = convection heat transfer coefficient

$T_s$ = Hot surface temperature

$T_\infty$ = Fluid temperature

$A$ = area of contact or exposure

Therefore, convection heat transfer can be increased by either of the following ways-

1. Increasing the temperature difference ($T_s - T_\infty$) between the surface and the fluid.

2. Increasing the convection heat transfer coefficient by enhancing the fluid flow or flow velocity over the body.

3. Increasing the area of contact or exposure between the surface and the fluid.

Most of the times, to control the temperature difference is not feasible and increase of heat transfer coefficient may require installation of a pump or a fan or replacing the existing one with a new one having higher capacity, the alternative is to increase the effective surface area by extended surfaces or fins.

**Fins** are the extended surface protruding from a surface or body and they are meant for increasing the heat transfer rate between the surface and the surrounding fluid by increasing heat transfer area.
Example of surfaces where fins are used

1. Air cooled I.C. engines
2. Refrigeration condenser tubes
3. Electric transformers
4. Reciprocating air compressors
5. Semiconductor devices
6. Automobile radiator

Types of fin

Fins can be broadly classified as:

1. Longitudinal fin
2. Radial fin
3. Pin fin

(a) Longitudinal fin – Rectangular profile
(b) Longitudinal fin – Rectangular profile
(c) Longitudinal fin - Trapezoidal profile
(d) Longitudinal fin - Concave parabolic
(e) Radial fin – Rectangular profile
(f) Radial fin – Triangular profile
(g) Pin fin – Cylindrical
(h) Pin fin – Tapered profile
(i) Pin fin – Concave parabolic

Analysis of fins with uniform cross sectional area

Rectangular fin

\[ T_0 = \text{base temperature or root temperature} \]

Heat is conducted from the base in to the fin at its root and then while simultaneously conducting along the length of the fin, heat is also convected from the surface of the fin to the ambient fluid with the convective heat transfer coefficient of \( h \) in W/m²-Kelvin.

Consider a differential element of the fin of length \( dx \). Let \( Q_x \) is the heat conducted in to the element along \( x \)-direction given by

\[ Q_x = -kA_c \frac{dT}{dx} \]  (from Fourier law of heat conduction) (1)

Where \( k = \text{thermal conductivity of fin material} \)
\[ A_c = \text{Area of cross section of the fin} \]
Let \( Q_{x+dx} \) = heat conducted out of the element along x-direction

\[
Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x) \, dx
\]  

(2)

\( Q_{\text{convected}} \) = heat transfer by convection from the surface of element to fluid

\[
Q_{\text{convected}} = h \,(A_{\text{conv}}) \,(T - T_\infty)
\]  

(3)

\( A_{\text{conv}} \) (convection area) = perimeter of fin \( \times \) length of element

\[
= P \times dx
\]

\( T \) = temperature of differential element

Assume steady state conditions and writing the energy balance equations for the element

Heat conducted in to the element = heat conducted out of the element + heat convected from the element to fluid

\[
Q_x = Q_{x+dx} + Q_{\text{convected}}
\]

\[
Q_x = Q_x + \frac{\partial}{\partial x}(Q_x) \, dx + h(A_{\text{conv}}) \,(T - T_\infty)
\]  

[ from equation 1,2 and 3 ]

\[
0 = \frac{\partial}{\partial x} (-kA_x \frac{dT}{dx}) dx + h(P \, dx) \,(T - T_\infty)
\]

Assuming \( k \) constant, we get

\[
\frac{d^2 t}{dx^2} - \frac{hP}{kA_C} \,(T - T_\infty) = 0
\]

Put \( T - T_\infty = \theta \) , then

\[
\frac{d\theta}{dx} = \frac{d\theta}{dx} \quad \text{and} \quad \frac{d^2 t}{dx^2} = \frac{d^2 \theta}{dx^2}
\]

Put \( \frac{hP}{kA_C} = m^2 \) , then

\[
\frac{d^2 \theta}{dx^2} - m^2 \theta = 0
\]

This is a standard format of 2\textsuperscript{nd} order differential equation in \( \theta \) whose general solution can be given as

\[
\theta = C_1 e^{-mx} + C_2 e^{mx}
\]
Where \( m = \sqrt{\frac{hP}{kA_c}} \)

And \( C_1 \) and \( C_2 \) are constant of integration that are to be obtained from boundary conditions.

**Note:**

For pin fin, the values of \( A_c \) and \( P \) will be different.

**Boundary conditions:**

(a) One common boundary condition is

At \( x = 0 \) (root), \( T = T_o \) and \( \theta = \theta_o = T_o - T_\infty \)

The other boundary condition i.e. at the tip depends upon three different cases which are as follows:

**Case -1: Fin is infinitely long (very long fin)**

When the fin is infinitely long then the temperature at the tip of the fin will be essentially that of the fluid
At \( x = \infty \), \( \theta = T - T_\infty = 0 \)

The general solution is of the form

\[ \theta = C_1 e^{-mx} + C_2 e^{mx} \]

when \( C_1 \to 0 \)

\[ \theta = C_2 e^{mx} \]

On applying boundary condition at root (\( x=0 \)), we get temperature distribution along fin as

\[ T = (T_o - T_\infty) e^{-mx} + T_\infty \]

The heat transfer through fin is:

\[ Q_{\text{fin}} = -kA_c \left( \frac{dT}{dx} \right)_{x=0} = \sqrt{hPkA_c} (T_o - T_\infty) \]

Case -2: Fin tip is insulated

When the fin tip is insulated then

Conduction heat transfer at \( x = L \) is equal to zero i.e. \( (-kA \frac{dT}{dx})_{x=L} = 0 \)

Hence, boundary condition will be
The temperature distribution along the length of the fin is given by:

\[
\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L-x)]}{\cosh(mL)}
\]

The resulting heat transfer rate through the fin will be

\[
Q_{\text{fin}} = -kA_c \left( \frac{dT}{dx} \right)_{x=0} = \sqrt{hPkA_c} \theta_0 \tanh mL
\]

**Case -3: Fin is finite in length and also loses heat by convection from its tip (End not insulated)**

Conduction heat transfer at \( x = L \) is equal to convection heat transfer from tip i.e.

\[
(-kA_c \frac{dT}{dx})_{x=L} = h(A_{\text{conv}})(T_{x=L} - T_\infty)
\]

Then the temperature distribution is given by

\[
\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh[m(L_c-x)]}{\cosh(mL_c)}
\]

and

Heat transfer rate through fin is

\[
Q_{\text{fin}} = \sqrt{hPkA_c} \theta_0 \tanh mL_c
\]
Where $L_c = \text{Corrected length}$

Corrected fin length $L_c$ is defined such that heat transfer from a fin of length $L_c$ with insulated tip is equal to heat transfer from the actual fin of length $L$ with convection at the fin tip.

For longitudinal fin (rectangular), $L_c = L + \frac{t}{2}$

For pin fin (cylindrical), $L_c = L + \frac{d}{4}$

**Fin Efficiency**

It is defined as the ration of actual heat transfer rate taking place through the fin and the maximum possible heat transfer rate that could occur through the fin i.e. when the entire fin is at its root temperature or base temperature.

The entire fin will be at its root temperature only when the material of the fin has infinite thermal conductivity.

\[
\eta_{\text{long fin}} = \frac{Q_{\text{act}}}{Q_{\text{max. possible}}} = \frac{\sqrt{h P k A_c}}{h (A_{\text{fin}}) (T_o - T_{\infty})} = \frac{1}{L} \sqrt{\frac{k A_c}{h P}} = \frac{1}{mL}
\]

\[
\eta_{\text{insulated tip}} = \frac{Q_{\text{act}}}{Q_{\text{max. possible}}} = \frac{\sqrt{h P k A_c}}{h (A_{\text{fin}}) (T_o - T_{\infty})} = \frac{\theta_o \tanh mL}{mL} = \frac{\tanh mL}{mL}
\]

where $A_{\text{fin}} = \text{convection heat transfer area of fin}$

$= \text{perimeter of fin} \times \text{length of fin}$
**Fin Effectiveness**

It is defined as the ratio between heat transfer rate with fin and the heat transfer rate without fin.

\[
\varepsilon_{\text{long fin}} = \frac{Q_{\text{fin}}}{Q_{\text{without fin}}} = \frac{\sqrt{hPka} (T_0 - T_\infty)}{h(A_F) (T_0 - T_\infty)} = \sqrt{\frac{kP}{hA_c}}
\]

\[
\varepsilon_{\text{insulated tip}} = \frac{Q_{\text{fin}}}{Q_{\text{without fin}}} = \frac{\sqrt{hPka} \theta_0 \text{Tanh mL}}{h(A_F) (T_0 - T_\infty)} = \text{Tanh mL} \sqrt{\frac{kP}{hA_c}}
\]

where, \(A_r\) area at the root equivalent to cross-sectional area of fin \((A_c)\).

When

(i) \(\varepsilon = 1\) : Fin does not affect the heat transfer at all.

(ii) \(\varepsilon < 1\) : Fin acts as insulation (if thermal conductivity \((k)\) of fin material is low).

(iii) \(\varepsilon > 1\) : Heat transfer will be increased.

Also,

1. \(\varepsilon_{\text{fin}} \propto \frac{1}{\sqrt{h}}\)

2. \(\varepsilon_{\text{fin}} \propto \sqrt{k}\)

3. \(\varepsilon_{\text{fin}} \propto \frac{P}{\sqrt{A_c}}\)

**Note:**

1. Fins are generally used where convection heat transfer coefficient \((h)\) values are relatively low i.e. when air or gas is the medium and heat transfer is by natural convection.
2. Fin material should be made of highly conductive materials. Aluminium is preferred: low cost and weight, resistance to corrosion

3. Lateral surface area i.e. \( \frac{P}{A_c} \) of the fin should be as high as possible.

4. Fins with parabolic and triangular profiles contain less material and are more efficient requiring minimum weight.

5. The efficiency of most fins used in practice is above 90 percent.

References

1. Fundamentals of Engineering Heat and Mass transfer by R C Sachdeva
2. Heat and Mass transfer by R K Rajput
3. NPTEL notes