Electromagnetic Theory

for

Electronics & Communication Engineering

By

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Syllabus for Electromagnetic Theory


Analysis of GATE Papers

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<tr>
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<th>Percentage of Marks</th>
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Module Test  
- Test Questions  
- Answer Keys & Explanations

Reference Books
Electromagnetic Field

Learning Objectives
After reading this chapter, you will know:
1. Elements of Vector Calculus
2. Operators, Curl, Divergence
3. Electromagnetic Coulomb's Law, Electric Field Intensity, Electric Dipole, Electric Flux Density
4. Gauss's Law, Electric Potential
5. Divergence of Current Density and Relaxation
6. Boundary Conditions
7. Biot-Savart's Law, Ampere Circuit Law, Continuity Equation
9. Faraday's Law, Motional EMF, Induced EMF Approach
10. Maxwell's Equations

Introduction
Cartesian coordinates \((x, y, z), -\infty < x < \infty, -\infty < y < \infty, -\infty < z < \infty\)
Cylindrical coordinates \((\rho, \phi, z), 0 \leq \rho < \infty, 0 \leq \phi < 2\pi, -\infty < z < \infty\)
Spherical coordinates \((r, \theta, \phi), 0 \leq r < \infty, 0 \leq \theta \leq \pi, 0 \leq \phi < 2\pi\)
Other valid alternative range of \(\theta\) and \(\phi\) are----
(i) \(0 \leq \theta < 2\pi, 0 \leq \phi \leq \pi\)
(ii) \(-\pi \leq \theta \leq \pi, 0 \leq \phi \leq \pi\)
(iii) \(-\pi/2 \leq \theta \leq \pi/2, 0 \leq \phi < 2\pi\)
(iv) \(0 < \theta \leq \pi, -\pi \leq \phi < \pi\)

Vector Calculus Formula

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Cartesian Coordinates</th>
<th>Cylindrical Coordinates</th>
<th>Spherical Coordinates</th>
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<tbody>
<tr>
<td>(a)</td>
<td>Differential Displacement (dl = dx a_x + dy a_y + dz a_z)</td>
<td>(dl = d\rho a_\rho + \rho d\phi a_\phi + dz a_z)</td>
<td>(dl = d\rho a_\rho + \rho d\phi a_\phi + d\rho\sin\theta d\phi a_\theta)</td>
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<tr>
<td>(b)</td>
<td>Differential Area (dS = dy dz a_y = dx dz a_x = dx dy a_z)</td>
<td>(dS = \rho d\phi dz a_\rho = d\rho dz a_\phi = \rho d\rho d\phi a_z)</td>
<td>(dS = r^2 \sin\theta d\theta d\phi a_r = r \sin\theta dr d\phi a_\theta = r dr d\theta a_\phi)</td>
</tr>
<tr>
<td>(c)</td>
<td>Differential Volume (dv = dx dy dz)</td>
<td>(dv = d\rho d\phi dz)</td>
<td>(dv = r^2 \sin\theta d\theta d\phi dr)</td>
</tr>
</tbody>
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Operators

1) \( \nabla V \) – Gradient, of a Scalar \( V \)
2) \( \nabla \cdot \vec{V} \) – Divergence, of a Vector \( \vec{V} \)
3) \( \nabla \times \vec{V} \) – Curl, of a Vector \( \vec{V} \)
4) \( \nabla^2 V \) – Laplacian, of a Scalar \( V \)

DEL Operator:

\[
\nabla = \frac{\partial}{\partial x} \hat{x} + \frac{\partial}{\partial y} \hat{y} + \frac{\partial}{\partial z} \hat{z} \text{ (Cartesian)}
\]

\[
= \frac{\partial}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial}{\partial \phi} \hat{\phi} + \frac{\partial}{\partial z} \hat{z} \text{ (Cylindrical)}
\]

\[
= \frac{\partial}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial}{\partial \theta} \hat{\theta} + \frac{1}{rsin \theta} \frac{\partial}{\partial \phi} \hat{\phi} \text{ (Spherical)}
\]

Gradient of a Scalar field

\( V \) is a vector that represents both the magnitude and the direction of maximum space rate of increase of \( V \).

\[
\nabla V = \frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \text{ For Cartesian Coordinates}
\]

\[
= \frac{\partial V}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial V}{\partial \phi} \hat{\phi} + \frac{\partial V}{\partial z} \hat{z} \text{ For Spherical Coordinates}
\]

\[
= \frac{\partial V}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial V}{\partial \theta} \hat{\theta} + \frac{1}{rsin \theta} \frac{\partial V}{\partial \phi} \hat{\phi} \text{ For Cylindrical Coordinates}
\]

The following are the fundamental properties of the gradient of a scalar field \( V \):

1. The magnitude of \( \nabla V \) equals the maximum rate of change in \( V \) per unit distance.
2. \( \nabla V \) points in the direction of the maximum rate of change in \( V \).
3. \( \nabla V \) at any point is perpendicular to the constant \( V \) surface that passes through that point.
4. If \( A = \nabla V \), \( V \) is said to be the scalar potential of \( A \).
5. The projection of \( \nabla V \) in the direction of a unit vector \( a \) is \( \nabla V \cdot a \) and is called the directional derivative of \( V \) along \( a \). This is the rate of change of \( V \) in direction of \( a \).

Example: Find the Gradient of the following scalar fields:

(a) \( V = e^{-z} \sin 2x \cosh y \)

(b) \( U = \rho^2 z \cos 2\phi \)

(c) \( W = 10r \sin^2 \theta \cos \phi \)

Solution:

(a) \( \nabla V = \frac{\partial V}{\partial x} \hat{x} + \frac{\partial V}{\partial y} \hat{y} + \frac{\partial V}{\partial z} \hat{z} \)

\[
= 2e^{-z} \cos 2x \cosh y \hat{x} + e^{-z} \sin 2x \sinh y \hat{y} - e^{-z} \sin 2x \cosh y \hat{z}
\]

(b) \( \nabla U = \frac{\partial U}{\partial \rho} \hat{\rho} + \frac{1}{\rho} \frac{\partial U}{\partial \phi} \hat{\phi} + \frac{\partial U}{\partial z} \hat{z} \)

\[
= 2\rho \cos 2\phi \hat{\rho} - 2\rho \sin 2\phi \hat{\phi} + \rho^2 \cos 2\phi \hat{z}
\]

(c) \( \nabla W = \frac{\partial W}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial W}{\partial \theta} \hat{\theta} + \frac{1}{rsin \theta} \frac{\partial W}{\partial \phi} \hat{\phi} \)

\[
= 10 \sin^2 \theta \cos \phi \hat{r} + 10 \sin 2\theta \cos \phi \hat{\theta} - 10 \sin \theta \sin \phi \hat{\phi}
\]
Divergence of a Vector

Statement: Divergence of $\vec{A}$ at a given point $P$ is the outward flux per unit volume as the volume shrinks about $P$.

Hence,

$$\text{Div} \ A = \nabla \cdot A = \lim_{\Delta v \to 0} \frac{\oint_S A \cdot ds}{\Delta v} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1)$$

Where, $\Delta v$ is the volume enclosed by the closed surface $S$ in which $P$ is located. Physically, we may regard the divergence of the vector field $A$ at a given point as a measure of how much the field diverges or emanates from that point.

$$\nabla \cdot A = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z} \quad \text{Cartesian System}$$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \quad \text{Cylindrical System}$$

$$= \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 A_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (A_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial A_\phi}{\partial \phi} \quad \text{Spherical System}$$

From equation (1),

$$\oint_S A \cdot dS = \int_V \nabla \cdot A \, dv$$

This is called divergence theorem which states that the total outward flux of the vector field $A$ through a closed surface $S$ is same as the volume integral of the divergence of $A$.

Example: Determine the divergence of these vector field

(a) $P = x^2yz a_x + xza_z$

(b) $Q = \rho \sin \phi \ a_\rho + \rho^2 a_\phi + z \cos \phi a_z$

(c) $T = \frac{1}{r^2} \cos \theta \ a_r + r \sin \theta \cos \phi \ a_\theta + \cos \theta a_\phi$

Solution:

(a) $\nabla \cdot P = \frac{\partial}{\partial x} (x^2yz) + \frac{\partial}{\partial y} (0) + \frac{\partial}{\partial z} (xz)$

$= 2xyz + x$

(b) $\nabla \cdot Q = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho Q_\rho) + \frac{1}{\rho} \frac{\partial}{\partial \phi} Q_\phi + \frac{\partial}{\partial z} Q_z$

$= \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho^2 \sin \phi) + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \theta} (\rho^2 \cos \phi) + \frac{1}{\rho \sin \theta} \frac{\partial}{\partial \phi} (\rho^2 z)$

$= 2 \sin \phi + \cos \phi$

(c) $\nabla \cdot T = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 T_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (T_\theta \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (T_\phi)$

$= \frac{1}{r^2} \frac{\partial}{\partial r} (\cos \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (r \sin^2 \theta \cos \phi) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} (\cos \theta)$

$= 0 + \frac{1}{r \sin \theta} 2r \sin \theta \cos \theta \cos \phi + 0$

$= 2 \cos \theta \cos \phi$
Curl of a Vector

Curl of a Vector field provides the maximum value of the circulation of the field per unit area and indicates the direction along which this maximum value occurs.

That is,

\[
\text{Curl} \ A = \nabla \times A = \lim_{\Delta S \to 0} \left( \oint A \cdot dl \right)_{\text{max}}
\]

\[
\nabla \times A = \begin{vmatrix}
\frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\
A_x & A_y & A_z
\end{vmatrix}
\]

\[
= \frac{1}{\rho} \begin{vmatrix}
\frac{\partial A_y}{\partial \phi} & \frac{\partial A_z}{\partial \rho} & \rho A_\phi \\
A_\rho & A_\phi & A_\theta
\end{vmatrix}
\]

\[
= \frac{1}{r^2 \sin \theta} \begin{vmatrix}
\frac{\partial A_\rho}{\partial r} & \frac{\partial A_\theta}{\partial \theta} & \frac{\partial A_\phi}{\partial \phi} \\
r A_\rho & r A_\theta & r \sin \theta A_\phi
\end{vmatrix}
\]

From equation (2) we may expect that

\[
\oint_L A \cdot dl = \iint_S (\nabla \times A) \cdot ds
\]

This is called stoke’s theorem, which states that the circulation of a vector field \( A \) around a (closed) path \( L \) is equal to the surface integral of the curl of \( A \) over the open surface \( S \) bounded by \( L \), Provided \( A \) and \( \Delta \times A \) are continuous no s.

Example: Determine the curl of each of the vector fields.

(a) \( P = x^2yz \ a_x + xza_z \)

(b) \( Q = \rho \sin \phi a_\rho + \rho^2 \ a_\phi + z \cos \phi a_z \)

(c) \( T = \frac{1}{r^2} \cos \theta \ a_r + r \sin \theta \cos \phi a_\theta + \cos \phi a_\phi \)

Solution:

(a) \( \nabla \times P = \left( \frac{\partial P_y}{\partial y} - \frac{\partial P_z}{\partial z} \right) a_x + \left( \frac{\partial P_z}{\partial x} - \frac{\partial P_x}{\partial x} \right) a_y + \left( \frac{\partial P_x}{\partial y} - \frac{\partial P_y}{\partial x} \right) a_z \)

\[
= (0 - 0) a_x + (x^2y - z) a_y + (0 - x^2z) a_z
\]

(b) \( \nabla \times Q = \left( \frac{1}{\rho} \frac{\partial Q_z}{\partial \phi} - \frac{\partial Q_\phi}{\partial z} \right) a_\rho + \left( \frac{\partial Q_\phi}{\partial z} - \frac{\partial Q_z}{\partial \phi} \right) a_\phi + \frac{1}{\rho} \left[ \frac{\partial}{\partial \rho} \left( \rho Q_\phi \right) - \frac{\partial Q_\rho}{\partial \phi} \right] a_z
\]

\[
= \left( \frac{-z}{\rho} \sin \phi - \rho^2 \right) a_\rho + (0 - 0) a_\phi + \frac{1}{\rho} \left( 3\rho^2z - \rho \cos \phi \right) a_z
\]

\[
= -\frac{1}{\rho} \left( z \sin \phi + \rho^3 \right) a_\rho + (3\rho z - \cos \phi) a_z
\]
(c) \( \nabla \times T = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (T_\phi \sin \theta) - \frac{\partial}{\partial \phi} T_\theta \right] a_r \)
\[ + \frac{1}{r} \left[ \frac{\partial}{\partial r} (r T_\phi) - \frac{\partial}{\partial \theta} (r T_\theta) \right] a_\phi \]
\[ = \frac{1}{r \sin \theta} \left[ \frac{\partial}{\partial \theta} (\cos \theta \sin \theta) - \frac{\partial}{\partial \phi} (r \sin \theta \cos \phi) \right] a_r \]
\[ + \frac{1}{r} \left[ \frac{1}{r \sin \theta \frac{\partial}{\partial \phi} (\cos \theta) - \frac{\partial}{\partial r} (r \sin \theta \cos \phi) \right] a_\phi \]
\[ = \frac{1}{r \sin \theta} \left[ r \sin \theta (\cos 2\theta + r \sin \theta \sin \phi) \right] a_r - \frac{\cos \theta}{r} a_\theta + (2 \cos \phi + \frac{1}{r^2}) \sin \theta a_\phi \]

**Laplacian**

(a) Laplacian of a scalar field \( V \), is the divergence of the gradient of \( V \) and is written as \( \nabla^2 V \).

\[ \nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} \rightarrow \text{For Cartesian Coordinates} \]

\[ \nabla^2 V = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial V}{\partial \rho} \right) + \frac{1}{r^2} \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 V}{\partial \phi^2} \rightarrow \text{For Cylindrical Coordinates} \]

\[ \nabla^2 V = \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial r} \left( r^2 \sin \theta \frac{\partial V}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial V}{\partial \theta} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial^2 V}{\partial \phi^2} \rightarrow \text{For Spherical Coordinates} \]

If \( \nabla^2 V = 0 \), \( V \) is said to be harmonic in the region.

A vector field is solenoidal if \( \nabla \cdot A = 0 \); it is irrotational or conservative if \( \nabla \times A = 0 \)

\( \nabla \cdot (\nabla \times A) = 0 \)

\( \nabla \times (\nabla V) = 0 \)

(b) Laplacian of Vector \( \vec{A} \)

\( \nabla^2 \vec{A} = \ldots \) is always a vector quantity

\( \nabla^2 \vec{A} = (\nabla^2 A_x) \hat{x} + (\nabla^2 A_y) \hat{y} + (\nabla^2 A_z) \hat{z} \)

\( \nabla^2 A_x \rightarrow \text{Scalar quantity} \)

\( \nabla^2 A_y \rightarrow \text{Scalar quantity} \)

\( \nabla^2 A_z \rightarrow \text{Scalar quantity} \)

\( \nabla^2 V = \frac{-\rho}{\varepsilon} \ldots \text{Poisson's E.q.} \)

\( \nabla^2 V = 0 \ldots \text{Laplace E.q.} \)

\( \nabla^2 E = \mu \sigma \frac{\partial E}{\partial t} + \mu E \frac{\partial^2 E}{\partial t^2} \ldots \text{wave E.q.} \)

**Example:** The potential (scalar) distribution in free space is given as \( V = 10y^4 + 20x^3 \).

If \( \varepsilon_0 \): permittivity of free space what is the charge density \( \rho \) at the point \( (2,0) \)?
Solution: Poisson's Equation $\nabla^2 V = -\frac{\rho}{\varepsilon}$

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)(10y^4 + 20x^3) = \frac{-\rho}{\varepsilon_0}$$

$\varepsilon = \varepsilon_r \varepsilon_0$ $[\varepsilon = \varepsilon_0$ as $\varepsilon_r = 1]$

$20 \times 3 \times 2 + 10 \times 4 \times 3y^2 = \frac{-\rho}{\varepsilon_0}$

At pt(2, 10) $\Rightarrow 20 \times 3 \times 2 \times 2 = \frac{-\rho}{\varepsilon_0}$

$\rho = -240\varepsilon_0$

Example: Find the Laplacian of the following scalar fields

(a) $V = e^{-z} \sin 2x \cosh y$

(b) $U = \rho^2 z \cos 2\phi$

(c) $W = 10r \sin^2 \theta \cos \phi$

Solution: The Laplacian in the Cartesian system can be found by taking the first derivative and later the second derivative.

(a) $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$$= \frac{\partial}{\partial x}(2e^{-z} \cos 2x \cosh y) + \frac{\partial}{\partial y}(e^{-z} \sin 2x \sinh y) + \frac{\partial}{\partial z}(-e^{-z} \sin 2x \cosh y)$$

$$= -4e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y + e^{-z} \sin 2x \cosh y$$

$$= -2e^{-z} \sin 2x \cosh y$$

(b) $\nabla^2 U = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho \frac{\partial U}{\partial \rho}) + \frac{\partial^2 U}{\partial \phi^2} + \frac{\partial^2 U}{\partial z^2}$

$$= \frac{1}{\rho} \frac{\partial}{\partial \rho}(2\rho^2 z \cos 2\phi) - \frac{1}{\rho^2} 4\rho^2 z \cos 2\phi + 0$$

$$= 4z \cos 2\phi - 4z \cos 2\phi$$

$$= 0$$

(c) $\nabla^2 W = \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \frac{\partial W}{\partial r}) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta}(\sin \theta \frac{\partial W}{\partial \theta}) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 W}{\partial \phi^2}$

$$= \frac{1}{r^2} \frac{\partial}{\partial r}(10r^2 \sin^2 \theta \cos \phi) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta}(10r \sin 2\theta \sin \theta \cos \phi) - \frac{10r \sin^2 \theta \cos \phi}{r^2 \sin^2 \theta} - \frac{10r \sin \theta \cos \phi}{r}$$

$$= \frac{20 \sin^2 \theta \cos \phi}{r} + \frac{20r \cos 2\theta \sin \theta \cos \phi}{r^2 \sin \theta} + \frac{10r \sin 2\theta \cos \theta \cos \phi}{r^2 \sin \theta}$$

$$= \frac{10 \cos \phi}{r} \left(2 \sin^2 \theta + 2 \cos 2\theta + 2 \cos^2 \theta - 1\right)$$

$$= \frac{10 \cos \phi}{r} \left(1 + 2 \cos 2\theta\right)$$

Stoke's Theorem

Statement: Closed line integral of any vector $\vec{A}$ integrated over any closed curve C is always equal to the surface integral of curl of vector $\vec{A}$ integrated over the surface area 's' which is enclosed by the closed curve 'c'.

Electromagnetic Field