Control Systems

For

EC / EE / IN

By

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Syllabus for Control Systems


Analysis of GATE Papers

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Learning Objectives

After reading this chapter, you will know:
1. Classification of Control Systems
2. Effect of Feedback
3. Block Diagram Reduction Techniques
4. Signal Flow Graphs

Introduction

It is a system by means of which any quantity of interest in a machine or mechanism is controlled (maintained or altered) in accordance with the desired manner. Following diagram depicts the block diagram representation of a control system.

![Block Diagram of a Control System]

Any system can be characterized mathematically by Transfer function or State model. Transfer function is defined as the ratio of Laplace Transform (L.T) of output to that of input assuming initial conditions to be zero. Transfer function is also obtained as Laplace transform of the impulse response of the system.

Transfer Function = \[
\frac{\text{Laplace transform of output}}{\text{Laplace transform of input}}\]

\[T(s) = \frac{L[c(t)]}{L[r(t)]} = \frac{C(s)}{R(s)}\]

For any arbitrary input \(r(t)\), output \(c(t)\) of control system can be obtained as below,

\[c(t) = L^{-1}[C(s)] = L^{-1}[T(s) R(s)] = L^{-1}(T(s) \ast r(s))\]

Where \(L\) and \(L^{-1}\) are forward and inverse Laplace transform operators and \(\ast\) is convolution operator.
Classification of Control Systems

Control systems can be classified based on presence of feedback as below,

1. **Open loop control systems**
2. **Closed loop control systems**

### Open-loop Control System

![Block Diagram of Open-Loop Control System](image)

- The reference input controls the output through a control action process. Here output has no effect on the control action, as the output is not fed-back for comparison with the input.
- Accuracy of an open-loop control system depends on the accuracy of input calibration.
- The open-loop system is simple and cheap to construct.
- Due to the absence of feedback path, the systems are generally stable.
- Examples of open loop control systems include Traffic lights, Fans, Washing machines etc, which do not have a sensor.
- If \( R(s) \) is LT of input and \( C(s) \) is LT of output of a control system of transfer function \( G(s) \), then \[
C(s) = G(s) R(s)
\]

### Closed-Loop Control System (Feedback Control Systems)

![Block Diagram of Closed Loop Control System](image)

- In a close-loop control system, the output has an effect on control action through a feedback.
- The control action is actuated by an error signal \( e(t) \) which is the difference between the input signal \( r(t) \) and the feedback signal \( f(t) \).
- The control systems can be manual or automatic control systems.
- Servomechanism is example of a closed-loop (feedback) control system using a power amplifying device prior to controller and the output of such a system is mechanical i.e., position, velocity or acceleration.

For Positive feedback, error signal \( e(t) = r(t) + f(t) \)
For Negative feedback, error signal \( e(t) = r(t) - f(t) \)
Transfer Function Representation of a Closed Loop Control System

Generally, the purpose of feedback is to reduce the error between the reference input and the system output.

Let $G(s)$ be the forward path transfer function, $H(s)$ be the feedback path transfer function and $T(s)$ be the overall transfer function of the closed-loop control system, then

$$T(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Here negative sign in denominator is considered for positive feedback and vice versa.

**Positive Feedback Control Systems**

- Unity Feedback ($H(s) = 1$) : $T(s) = \frac{G(s)}{1 - G(s)}$
- Non Unity Feedback ($H(s) \neq 1$) : $T(s) = \frac{G(s)}{1 - G(s)H(s)}$

**Negative Feedback Control Systems**

- Unity F/B : $T(s) = \frac{G(s)}{1 + G(s)}$
- Non Unity F/B : $T(s) = \frac{G(s)}{1 + G(s)H(s)}$

Here, $G(s)$ is T.F. without feedback (or) T. F of the forward path and $H(s)$ is T.F. of the feedback path. Block diagram shown below corresponds to closed loop control system.

The overall transfer function can be derived as below,

$L\{r(t)\} = R(s) \rightarrow$ Reference input
$L\{c(t)\} = C(s) \rightarrow$ Output (Controlled variable)
$L\{f(t)\} = F(s) \rightarrow$ Feedback signal
$L\{e(t)\} = E(s) \rightarrow$ Error or actuating signal
$G(s)H(s) \rightarrow$ Open loop transfer function
E(s)/R(s) $\rightarrow$ Error transfer function
G(s) $\rightarrow$ Forward path transfer function
H(s) $\rightarrow$ Feedback path transfer function
\[
E(s) = R(s) \pm F(s); F(s) = H(s) C(s)
\]
\[
C(s) = \frac{E(s) G(s)}{R(s)} = \frac{C(s)}{R(s)} = \frac{\{ R(s) \pm H(s)C(s) \} G(s)}{1 \pm G(s)H(s)}
\]
Also,
\[
\frac{E(s)}{R(s)} = \frac{1}{1 \pm G(s)H(s)}
\]
Here negative sign is used for positive feedback and positive sign is used for negative feedback.
The transfer function of a system depends upon its elements assuming initial conditions as zero and it is independent of input function.

**Comparison of Open-Loop and Close-Loop Control Systems**
Table below summarizes the comparison between open and closed loop control systems.

<table>
<thead>
<tr>
<th>Sl. No</th>
<th>Open-Loop C.S.</th>
<th>Closed-Loop C.S.</th>
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<tbody>
<tr>
<td>1.</td>
<td>The accuracy of an open-loop system depends on the calibration of the input. Any departure from pre-determined calibration affects the output.</td>
<td>As the error between the reference input and the output is continuously measured through feedback, the closed-loop system works more accurately.</td>
</tr>
<tr>
<td>2.</td>
<td>The open-loop system is simple to construct.</td>
<td>The close-loop system is complicated to construct.</td>
</tr>
<tr>
<td>4.</td>
<td>The open-loop systems are generally stable.</td>
<td>The close-loop systems can become unstable under certain conditions.</td>
</tr>
<tr>
<td>5.</td>
<td>The operation of open-loop system is affected due to presence of non-linearities in its elements.</td>
<td>In terms of the performance, the closed-loop systems adjusts to the effects of non-linearities present in its elements.</td>
</tr>
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**Effects of Feedback**
The feedback has effects on system performance characteristics such as stability, bandwidth, overall gain, impedance and sensitivity.

1. **Effect of Feedback on Stability**
   - Stability is a notion that describes whether the system will be able to follow the input command.
   - A system is said to be unstable, if its output is out of control or increases without bound. For a bounded input.
     - If the input itself is not bounded, then the output would definitely increase without bound, even if the system is inherently stable.
   - Negative feedback in a control system introduces a possibility of instability, if not properly tuned.

2. **Effect of Feedback on Overall Gain**
   - Negative feedback decreases the gain of the system and Positive feedback increases the gain of the system.
3. **Effect of Feedback on Sensitivity**

Consider $G$ as a parameter that can vary. The sensitivity of the gain of the overall system $T$ to the variation in $G$ is defined as

$$S_G^T = \frac{\partial T}{\partial G} \frac{T}{G} = \% \text{ change in } T \div \% \text{ change in } G$$

Where $\partial T$ denotes the incremental change in $T$ due to the incremental change in $G$; $\partial T/T$ and $\partial G/G$ denote the percentage change in $T$ and $G$, respectively.

$$S_G^T = \frac{\partial T}{\partial G} = \frac{1}{1 + GH}$$

Similarly, $S_H^T = \frac{\partial T}{\partial H} = \frac{-GH}{1 + GH} \approx -1$

In general, the sensitivity of the system gain of a feedback system to parameter variation depends on where the parameter is located.

$$S_G^T = \frac{1}{1 + GH} \quad \text{and} \quad S_H^T = \frac{-GH}{1 + GH} \approx -1$$

Therefore, an increase in the value of “GH” tends to reduce the sensitivity of the system gain to variations in the parameter “G”. But the same increase in “GH” produces a one-to-one response to variations in “H”. A decrease in the value of “GH” would reduce the sensitivity of the system gain to variations in the parameter “H”.

4. Negative feedback improves the dynamic response of the system.
5. Negative feedback reduces the effect of disturbance signal or noise.
6. Negative feedback improves the Bandwidth of the system.
7. Negative feedback improves the accuracy the system by reducing steady state error.
8. Improved rejection of disturbances.

Let $\alpha =$ A variable that changes its value

$\beta = $ A parameter that changes the value of $\alpha$

$$S_{\beta}^\alpha = \frac{\% \text{ change in } \alpha}{\% \text{ change in } \beta} = \frac{\partial \alpha}{\partial \beta} = \alpha \times \frac{\partial}{\partial \beta}$$

**Open Loop Control System**

$$R(s) \xrightarrow{G(s)} C(s)$$

$\alpha = T(s) [\text{open loop control system}] = \frac{C(s)}{R(s)} = G(s)$

$\beta = G(s)$

$$S_{G(s)}^{T(s)} = \frac{G(s)}{T(s)} \times \frac{2T(s)}{2G(s)} = 1 \times 1 = 1 \space \therefore \alpha = \beta$$

**Closed Loop Control System**

$$R(s) \xrightarrow{G(s)} C(s) \xrightarrow{H(s)}$$