Signals and Systems

Answer Keys and Explanations

1. **[Ans. *] Range: 3.9 to 4.1**

$x_1(n) = 2^n\, u(n)$; $x_2(n) = 3^n u(n)$

$x(n) = 3x_1(n) - 4x_2(n); x(z) = 3x_1(z) - 4x_2(z)$

$x_1(z) = \frac{1}{1-2z^{-1}}$ $|z| > 2$

$x_2(z) = \frac{1}{1-3z^{-1}}$ $|z| > 3$

$x(z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}}$ $|z| > 3$

$= \frac{3z}{z-2} - \frac{4z}{z-3} = -\frac{z^2+z}{z^2-5z+6}$

$\therefore a = 1; b = 1; c = 1; d = -5, e = 6$

$\therefore a + b + c + d + e = 4$

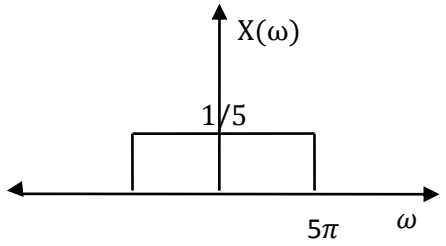
2. **[Ans. *] Range: 0.2 to 0.2**

$x(t) = \sin c(5t)$

$x(t) = Sa(5\pi t)$

Energy $E = \int_{-\infty}^{\infty} x^2(t)dt$

Using Parseval Theorem: $E = \int_{-\infty}^{\infty} x^2(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^2(\omega)d\omega$



$E = \frac{1}{2\pi}\int_{-\infty}^{\infty} X^2(\omega)d\omega$

$= \frac{1}{2\pi}\int_{-5\pi}^{5\pi} (\frac{1}{5})^2 d\omega = \frac{1}{5} = 0.2$

3. **[Ans. D]**

$f(t) = 2u(t) + r(t) - r(t-1) - u(t-1) - 2r(t-3) + 2r(t-4)$

$f(s) = \frac{2}{s} + \frac{1}{s^2} - \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s} - \frac{2e^{-3s}}{s^2} + \frac{2e^{-4s}}{s^2}$

$= \frac{1}{s}[2 - e^{-s}] + \frac{1}{s^2}[1 - e^{-s} - 2e^{-3s} + 2e^{-4s}]$

4. **[Ans. A]**

$$f(t) = 2r(t) - 2r\left(t - \frac{1}{2}\right) - 2r\left(t - \frac{1}{2}\right) + 2r(t-1)$$
$$= 2r(t) - 4r\left(t - \frac{1}{2}\right) + 2r(t-1)$$
$$f(s) = \frac{2}{s^2} - \frac{4e^{-0.5s}}{s^2} + 2\frac{e^{-s}}{s^2}$$
$$= \frac{2}{s^2}[1 - 2e^{-0.5s} + e^{-s}]$$
$$= \frac{2}{s^2}(1 - e^{-0.5s})^2$$

5. **[Ans. B]**

$e^{-\pi t^2} \xrightarrow{F.T} e^{-\pi f^2}$; $e^{-\alpha t^2} = e^{-\pi\left(\sqrt{\frac{\alpha}{\pi}}\, t\right)^2}$

By applying property of F.T pair

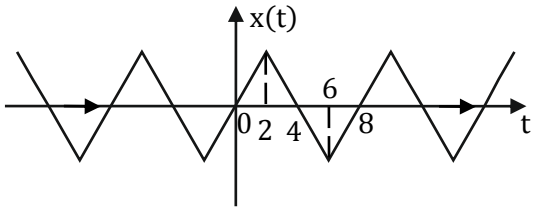
If $x(t) \xrightarrow{F.\,T} X(j\omega)$

Then $x(at) \xrightarrow{F.T} \frac{1}{|a|} X\left(\frac{j\omega}{a}\right)$

So, $e^{-\pi\left(\sqrt{\frac{\alpha}{\pi}}\, t\right)^2} \xrightarrow{F.\,T} \sqrt{\frac{\pi}{\alpha}}\; e^{-\pi\, f^2\left(\frac{\pi}{\alpha}\right)}$

$= \sqrt{\frac{\pi}{\alpha}}\; e^{-\frac{\pi^2 f^2}{\alpha}}$

6. **[Ans. B]**



⇒ x(t) is odd signal and also half ware symmetry signal

⇒ F.S. expression will contain only

Odd harmonics of sine term (B)

7. **[Ans. *] Range 1370 to 1370**

$x(n) = 2\,\delta(n) + \delta(n-1) + 3\delta(n-2) = \{2, 1, 3\}$

$h(n) = 3\delta(n) + x\delta(n-1) + y\delta(n-2) = \{8, x, y\}$

$y(n) = x(n) * h(n)$

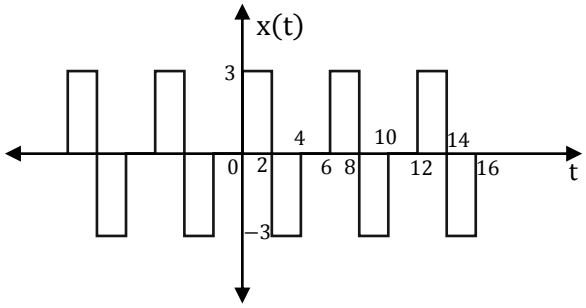
$y(1) = 6$

$y(2) = 3$

	↓		
	2	1	3
→ 8	16	8	24
X	2x	x	3x
Y	2y	y	3y

$y(n) = \{16, 2x + 8, 2y + x + 24, y + 3x, 3y\}$
$y(1) = 2x + 8 = 6 \rightarrow x = -1$
$y(2) = 2y + x + 24 = 3 \rightarrow y = -10$
$y(n) = \{16, 6, 3, -13, -30\}$
$E = \sum_{n=0}^{4} (y(n)^2 = [(16)^2 + (6)^2 + (3)^2 + (-13)^2 + (-30)^2] = 1370$

8. **[Ans. *] Range 6 to 6**



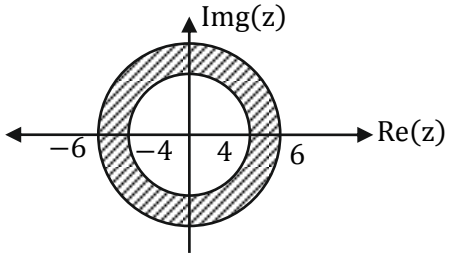
⇒ Average power is independent to time shifting as well as time scaling
⇒ So average power of y(t) is equal to x(t)

$P_{av} = \frac{1}{6}\int_0^6 1x(t)^2\, dt$

$= \frac{1}{6}[9 \times 2 + 9 \times 2] = \frac{1}{6}[18 + 18] = \frac{36}{6} = 6$

9. **[Ans. D]**

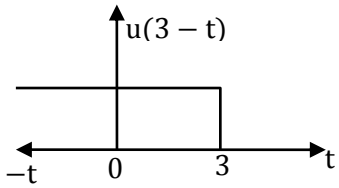
$x(n) = 3\left[-\frac{1}{0.25}\right]^n u(n) - 8[6]^4\, u(-n-1)$
$x(n) = 3[-4]^n\, u(n) - 8\,[6]^n\, u(-n-1)$
ROC $|z| > 4$ $|z| < 6$
$4 < |z| < 6$ i.e $(0.25)^{-1} < z < \left(\frac{1}{6}\right)^{-1}$



Option (D) is correct

10. **[Ans. C]**

$h(t) = e^{+5t}\, u(3-t)$
for $u(3-t) \rightarrow$ Indigatest triet limituy from -8 to $+3$



$\Rightarrow h(t)$ has a finite value for all values of $t[-\infty < t < 3]$, so it is stable

$\Rightarrow h(t) \neq 0, t < 0 \rightarrow \text{Non} - \text{causual}$

Option (C) (Stable and Non-causal)