Part – 7: Measurements

7.1: Basics of Measurements and Error Analysis (Static & Dynamic characteristics of measuring Instrument)

Fundamental Units: These include the following along with dimension and unit symbol.

Mass	M	Kg
Length	L	m
Time	T	Sec
Temperature	θ	K
Electric Current	I	A
Luminous Intensity		Cd

Performance Characteristics

- The performance characteristics of an instrument are mainly divided into two categories:
  1. Static characteristics
  2. Dynamic characteristics
- Set of criteria defined for the measurements, which are used to measure the quantities, which are slowly varying with time or almost constant, i,e do not vary with time, are called **static characteristics**
- When the quantity under measurement changes rapidly with time, the relation existing between input and output are generally expressed with the help of differential equations and are called **dynamic characteristics**
- The various performance characteristics are obtained in one form or another by a process called **calibration**

Static Characteristics

1. **Accuracy:** It is the degree of closeness with which the instrument reading approaches the true value of the quantity.

2. **Static error**: It is the difference between the measured value and true value of the quantity
   Mathematically
   $\delta A = A_m - A_t$ ----------- eq (1.1)
   δA : absolute static error
   A_m: Measured value of the quantity.
   A_t : True value of the quantity.
   Relative error: $(\epsilon_r) = \frac{\delta A}{A_t}$
   $\epsilon_r = \frac{A_m - A_t}{A_t}$

Percentage relative error: $\% \in_r = \left(\frac{\delta A}{A_t}\right) \times 100.$

From relative percentage error, accuracy is expressed as $A = 1 - |\in_r|$

Where A: relative accuracy and $a = A \times 100\%$

where a = percentage Accuracy.

- error can also be expressed as percentage of full scale reading (FSD) as,
  $= \frac{A_m - A_t}{FSD} \times 100$

3. **Precision:** It is the measure of degree of agreement within a group of measurements.

- High degree of precision does not guarantee accuracy.
  Precision is composed of two characteristics
  1. Conformity.
  2. Number of significant figures.

4. **Significant Figures**

- Precision of the measurement is obtained from the number of significant figures, in which the reading is expressed.
- Significant figures convey the actual information about the magnitude and measurement precision of the quantity.

5. **Sensitivity**

The sensitivity denotes the smallest change in the measured variable to which the instrument responds.

It is defined as the ratio of the changes in the output of an instrument to a change in the value of the quantity to be measured.

Mathematically it is expressed as,

$\text{Sensitivity} = \frac{\text{Infinitesimal change in output}}{\text{Infinitesimal change in input}}$

∴ $\text{Sensitivity} = \frac{\Delta q_o}{\Delta q_i}$

∴ $\text{Deflection factor} = \frac{1}{\text{Sensitivity}} = \frac{\Delta q_i}{\Delta q_o}$

6. **Resolution**

Resolution is the smallest measurable input change.

7. **Threshold**

If the input quantity is slowly varied from zero onwards, the output does not change until some minimum value of the input is exceeded. This minimum value of the input is called **threshold.**

Resolution is the smallest measurable **input change** while the threshold is the smallest measurable **input.**

8. Linearity

Linearity is the ability to reproduce the input characteristics symmetrically and linearly. Graphically such relationship between input and output is represented by a straight line.
The graph of output against the input is called the **calibration curve.**
The linearity property indicates the straight line nature of the calibration curve.

Thus, the linearity is defined as,

$$\% \text{ Linearity} = \frac{\text{Maximum deviation of output from idealized straight line}}{\text{Full scale deflection}} \times 100$$

9. Zero Drift

The **drift** is the gradual shift of the instrument indication, over an extended period during which the value of the input variable does not change.

10. Reproducibility

It is the degree of closeness with which a given value may be repeatedly measured. It may be specified interms of units for a given period of time.

11. Repeatability: Repeatability is defined as variation of scale reading and is random in nature. Both reproducibility and the repeatability are a measure of the closeness with which a given input may be measured again and again. The Fig shows the input and output relationship with positive and negative repeatability.

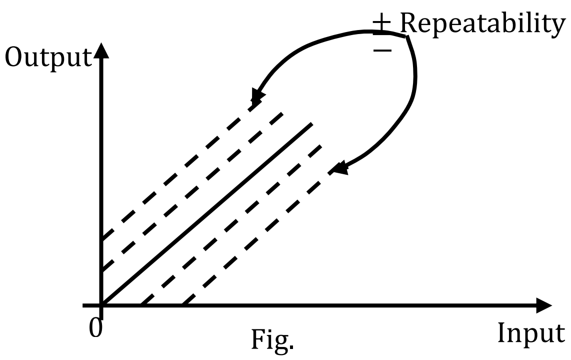


Fig.

12. Stability

The ability of an instrument to retain its performance throughout its specified operating life and the storage life is defined as its **stability.**

13. Tolerance:

The maximum allowable error in the measurement is specified interms of some value which is called **tolerance.** This is closely related to the accuracy.

14. Range or Span

The minimum and maximum values of a quantity for which an instrument is designed to measure is called its **range** or **span.** Sometimes the accuracy is specified interms of range or span of an instrument.

Limiting Errors/ Relative Limiting Error

Guarantee Errors: The limits of deviations from the specified value are defined as **limiting errors or guarantee errors.**

Actual value of quantity = A= An ± δa; δa: limiting error or tolerance

An: specified or rated value

- It is also called as fractional error. It is the ratio of the error to the specified magnitude of a quantity.

$$e = \frac{\delta A}{A_{sn}}$$

e → Relative limiting error.

Combination of Quantities with Limiting Errors

1. **Sum of the Two Quantities:** Let a_1 and a_2 be the two quantities which are to be added to obtain the result as A_T.

∴
$$e_T = \pm\left[\frac{a_1}{A_T}.e_1 + \frac{a_2}{A_T}.e_2\right]$$

Where $e_1 = \frac{\delta a_1}{a_1}$ and $e_2 = \frac{\delta a_2}{a_2}$ and $e_T = \frac{\delta A}{A_e}$

2. **Difference of the Two Quantities**

$$e_T = \pm\left[\frac{a_1}{A_T}.e_1 + \frac{a_2}{A_T}.e_2\right]$$

3. **Product of the Two Quantities**

$$e_T = \pm(e_1 + e_2)$$

4. **Division of the Two Quantities**

$$e_T = \pm(e_1 + e_2)$$

$$e_T = \pm[e_1 + e_2 + e_3 + \ldots]$$

5. **Power of a factor**

$$e_T = \pm n\, e_1$$

$$e_T = \pm[ne_1 + m\,e_2]$$

Types of Errors

The static error may arise due to number of reasons. The static errors are classified as

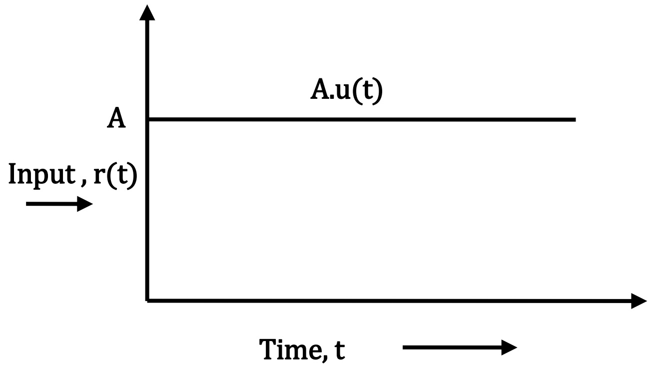
1. Gross errors
2. Systematic errors
3. Random errors

Time Domain test signals

1. Step input

$$R(s) = \int_0^\infty Ae^{-st}dt$$

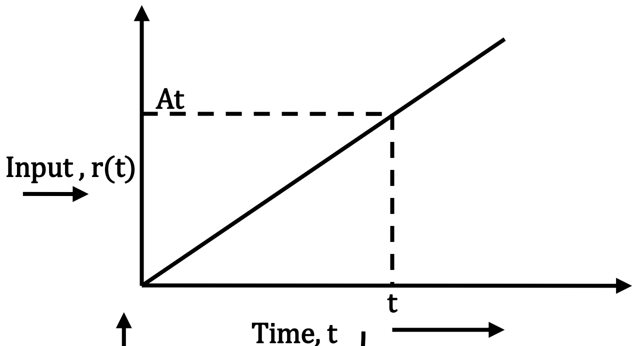
$$= A/S$$



2. Ramp input

$$R(s) = \int_0^\infty Ate^{-st}dt$$

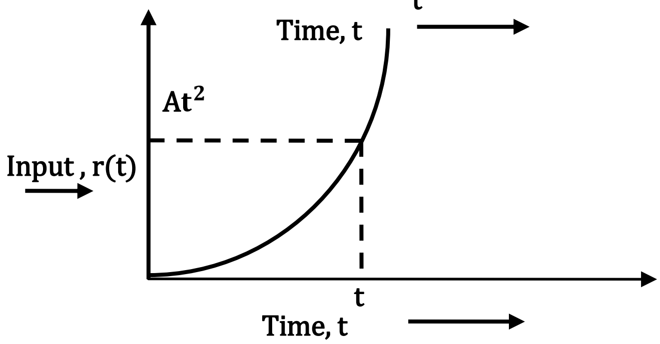
$$= A/S^2$$



3. Parabolic input

$$R(s) = \int_0^\infty At^2e^{-st}dt$$

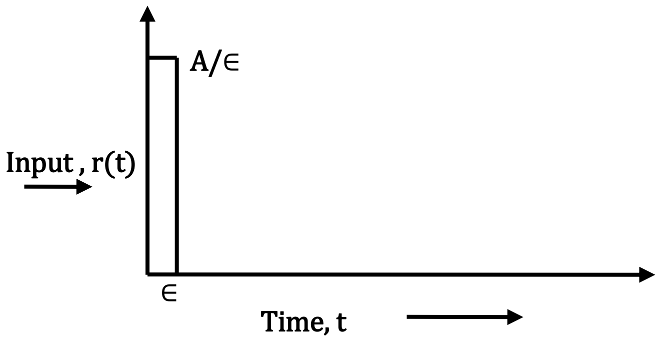
$$= 2A/S^3$$



4. Impulse input

$$R(s) = \int_0^\infty \frac{u(t)-u(t-\epsilon)}{\epsilon} e^{-st}dt$$

$$= 1$$



Response of First Order System to a Unit Step Input

$$C(t) = 1 - e^{-t/\tau}$$

$$e_m(t) = e^{-t/\tau}$$

$$e_{ss} = \lim_{t\to\infty} e_m(t) = 0$$

Ramp Response of a First Order System

$$C(t) = 1 - \tau[1 - e^{-t/\tau}]$$

$$e_m = \tau[1 - e^{-t/\tau}]$$

$$e_{ss} = \tau$$

Impulse Response of a First Order System

$$C(t) = \frac{A}{\tau}[e^{-t/\tau}]$$