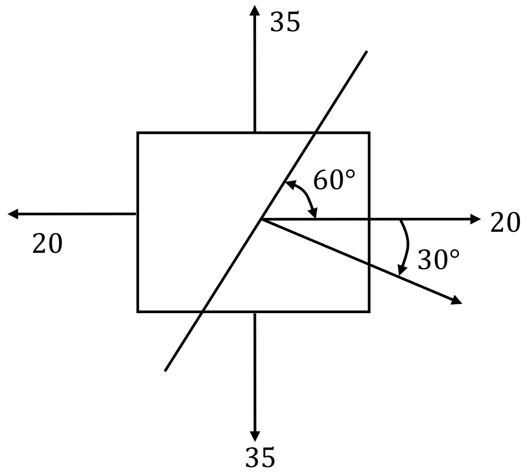
Analysis of stress and strain
Answer Keys and Explanations

1. **[Ans. D]**
In simple tension case
$\sigma_{max} = \sigma_x = P/A$
$\tau_{max} = \left(\frac{\sigma_x}{2}\right)$
$\Rightarrow \sigma_{max} = 50$ MPa

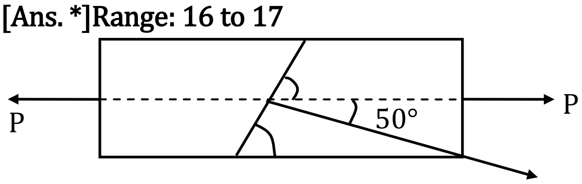
2. **[Ans. *]Range: 23 to 25**



Hence $\theta = -30°$
$\sigma_{-30} = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \tau_{xy}\sin 2\theta$
$\sigma_{-30} = (27.5) + (-7.5)(0.5)$
$\sigma_{-30} = 23.75$
$\tau_{-30} = -\left(\frac{\sigma_x - \sigma_y}{2}\right)\sin 2\theta + \tau_{xy}\cos 2\theta$
$= -(-7.5)\sin 60 = 6.49$
$\sigma_{net} = \sqrt{23.75^2 + 6.49^2}$
$= 24.62$

3. **[Ans. C]**
The above case is a pure shear loading condition

4. **[Ans. *]Range: 16 to 17**



$v = 50°$
$\sigma_\theta = \frac{P}{A}\cos^2\theta$

$= \left(\frac{200 \times 10^3}{50 \times 100} \times \cos^2 50\right)$

$= 16.52$ MPa

5. **[Ans. A]**

For hydrostatic state of stress

$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} -P & 0 \\ 0 & -P \end{bmatrix}$

Center of Mohr's circle

$= \left(\frac{\sigma_x + \sigma_y}{2}, 0\right)$

$= (-P, 0)$

Radius of Mohr's circle

$= \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

$= 0$

6. **[Ans. D]**

$\sigma_x = \sigma$

$\sigma_y = -\sigma$

$\tau_{xy} = 0$

$\Rightarrow \sigma_1 = \sigma$

$\sigma_2 = -\sigma$

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + (\tau_{xy})^2}$

$= \sigma$

7. **[Ans. *]Range: 50 to 50**

$\sigma_1 = 100$

$\sigma_2 = 50$

$\sigma_3 = 0$

$\tau_{max} = \left(\frac{\sigma_1 - \sigma_3}{2}\right)$

$= \frac{100 - 0}{2}$

$= 50$ MPa

8. **[Ans. *]Range: 15 to 15**

Normal stress σ_n maximum shear stress plane

$= \left(\frac{\sigma_x + \sigma_y}{2}\right)$

$=15$ MPa

9. **[Ans. B]**

$\sigma_x = -6$
$\sigma_y = 4$
$\tau_{xy} = -8$

$$\sigma_{1,2} = \left(\frac{-6+4}{2}\right) \pm \sqrt{\left(\frac{-6-4}{2}\right)^2 + (-8)^2}$$

$= -1 - \sqrt{(-5)^2 + (-8)^2}$
$= -10.43$ MPa

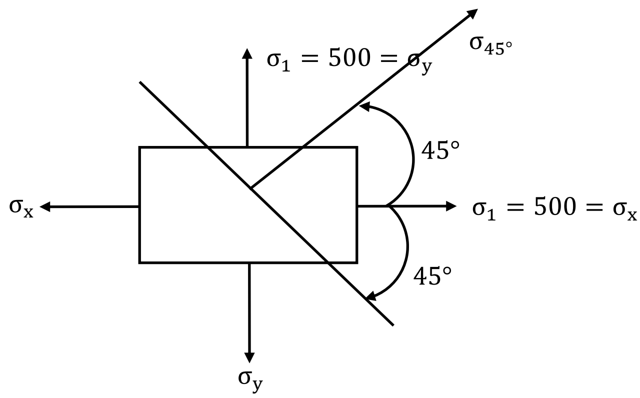
10. **[Ans. *]Range: 0.1 to 0.1**

$\in_0 = 1000 \times 10^{-4} = \in_x$
$\in_{45} = 800 \times 10^{-4}$
$\in_{90} = 600 \times 10^{-4} = \in_y$

$$\left(\frac{\tau_{xy}}{2}\right) = \in_{45} - \left(\in_0 + \frac{\in_{90}}{2}\right)$$

$\tau_{xy} = 0$
$\in_1 = (800 \times 10^{-4}) + \sqrt{(200 \times 10^{-4})^2}$
$\in_1 = 1000 \times 10^{-4} = 0.1$

11. **[Ans. B]**



$$\sigma_\theta = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \left(\frac{\sigma_x - \sigma_y}{2}\right)\cos 2\theta + \left(\tau_{xy}\right)\sin 2\theta$$

$\sigma_{45} = (500) + (0) + 0$
$\sigma_{45^\circ} = 500$

12. **[Ans. *]Range: 19 to 20**

$$\sigma_1 = \left(\frac{\sigma_x + \sigma_y}{2}\right) + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \left(\tau_{xy}\right)^2}$$

$$\sigma_1 = \left(\frac{20+10}{2}\right) + \sqrt{\left(\frac{20-10}{2}\right)^2 + (5)^2}$$

$\sigma_1 = 15 + \sqrt{5^2 + 5^2} = 22.07$
$\sigma_2 = 15 - \sqrt{5^2 + 5^2} = 7.93$
According to von mises criterion

$$S_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+(\sigma_2+\sigma_3)^2+(\sigma_1-\sigma_3)^2\right.}$$

For plane stress condition σ_3 won't exist

$$\text{So, } S_{yt} = \sqrt{\frac{1}{2}\left[(\sigma_1-\sigma_2)^2+\sigma_2^2+\sigma_1^2\ \right]}$$

$= 19.36$ MPa

13. **[Ans. B]**

Given, $\sigma_x = 20$ MPa

$\sigma_y = 10$ MPa

$\tau_{xy} = 5$MPa

$$\epsilon_x = \frac{1}{E}\left[\sigma_x - \mu\,\sigma_y\right] \Rightarrow \epsilon_x = 8.5\times10^{-5}$$

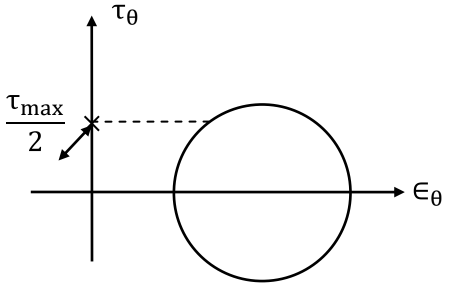
$$\epsilon_y = \frac{1}{E}\left[\sigma_y - \mu\,\sigma_x\right] \Rightarrow \epsilon_y = 2\times10^{-5}$$

$$\epsilon_z = \frac{1}{E}\left[-\mu(\sigma_x+\sigma_y)\right] \Rightarrow \epsilon_z = -4.5\times10^{-5}$$

$$\tau_{xy} = \left(\frac{\tau_{xy}}{G}\right) = \frac{(\tau_{xy})2(1+\mu)}{E} = 6.5\times10^{-5}$$

$$\text{Hence, t is } \begin{bmatrix} 8.5 & 6.5 & 0 \\ 6.5 & 2 & 0 \\ 0 & 0 & -4.5 \end{bmatrix}\times10^{-5}$$

14. **[Ans. *]Range: 0.5 to 0.9**



$$\frac{\tau_{max}}{2} = \text{Radius}$$

$\Rightarrow \tau_{max} = $ Diameter

$$\text{but, }\left[\frac{\tau_{max}}{2}\right] = \left[\frac{\epsilon_1-\epsilon_2}{2}\right]$$

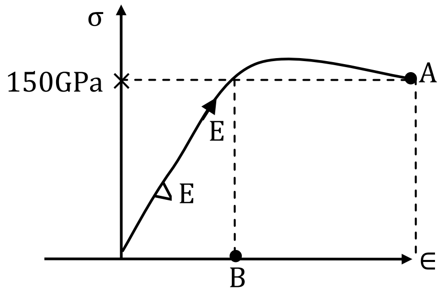
$\Rightarrow \tau_{max} = \epsilon_1 - \epsilon_2$

Hence diameter $= (\epsilon_1-\epsilon_2) = 0.7$

15. **[Ans. B]**

Brittle materials are based on max principle stress theory. It is also called as rankines theory

16. **[Ans. D]**



Assuming Hooks law is valid

$\frac{\sigma}{\epsilon} = E$

$\epsilon = \left(\frac{\sigma}{E}\right) = \left(\frac{150}{200}\right) = 0.75$

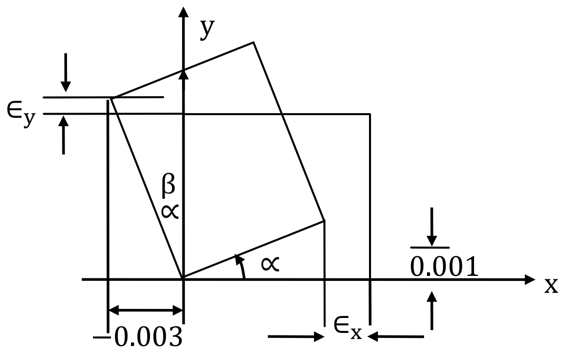
If, strain corresponding to B is 0.75 than, strain corresponding to A should be higher than 0.75

Hence its 0.8

17. **[Ans. *]Range: 0.002 to 0.003**

Hence $\epsilon_x = 0.002$

$\epsilon_y = 0.0025$



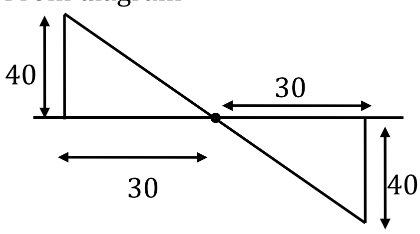
$\tau_{xy} = (\alpha + \rho)$

$= \left(\frac{0.001}{1}\right) + (-0.003) = -0.002$

Hence, $\epsilon_x + \epsilon_y + \tau_{xy} = 0.0025$

18. **[Ans. A]**

From diagram

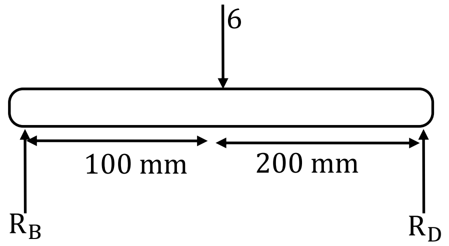


$\sigma_A = -50$

$\sigma_B = 30 - 20$

$= 10$

19. **[Ans. *] Range: 0.13 to 0.19**



$\sum F_y = 0 \Rightarrow R_B + R_D = 6kN$

$\sum M = 0 \Rightarrow -6(0.1) + R_D(0.3) = 0 \Rightarrow R_b = 2kN$

$\Rightarrow R_B = 4kN$

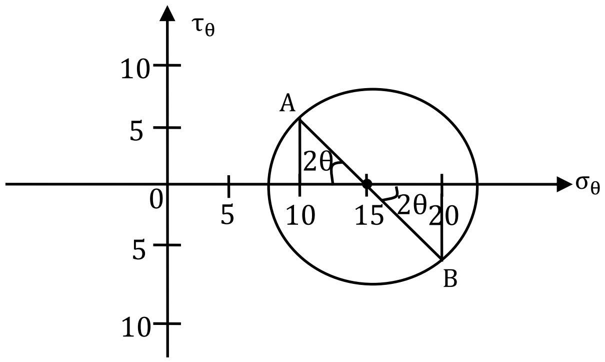
Elongation of AB $= \left(\frac{R_B L}{AE}\right)$

$= 0.16$ mm

20. **[Ans. *]Range: 22 to 23**

$A \rightarrow (10, 5)$

$B \rightarrow (20, -5)$



$\tan 2\theta = \left(\frac{5}{5}\right) = 1$

$\Rightarrow v = \left(\frac{45°}{2}\right) = 22.5°$