

$$y_1^{(C)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1)] \dots \dots \text{corrector Method}$$

$$\text{Predictor corrector: } y_1^{(PC)} = y_0 + \frac{h}{2} [f(x_0, y_0) + f(x_1, y_1^{(1)})]$$

$$\text{As } y_0 = 1 \& x_0 = 0, x_1 = x_0 + h = 0.3$$

$$y_1^{(p)} = 1 + 0.3(-2 \times (0) \times 1) = 1$$

$$y_1^{(C)} = 1 + \frac{0.3}{2} [0 + (-2)(0.3)^2(1)]$$

$$= 0.973$$

$$y_1^{(P-C)} = 1 + \frac{0.3}{2} [0 + (-2)(0.3)^2(0.973)]$$

$$= 0.973729$$

18. [Ans. *] Range: 0.66 to 0.67

Error in three equations is

$$\epsilon_1 = 3x - 2$$

$$\epsilon_2 = 2x - 1$$

$$\epsilon_3 = 4x - 3$$

$f(x)$ = sum of square errors

$$= \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2$$

$$= (3x - 2)^2 + (2x - 1)^2 + (4x - 3)^2$$

For minimum value of sum of squares

$$f'(x) = 0$$

$$f''(x) > 0$$

$$f'(x) = 2(3x - 2) + 2(2x - 1) + 2(4x - 3) = 0$$

$$9x - 6 = 0$$

$$x = \frac{2}{3}$$

$$f''(x) = 2(3) + 2(2) + 2(4) = 26 > 0$$

$\therefore f(x)$ is minimum at $x = \frac{2}{3}$

$$x^3 - 6x^2 - 13x + 42 = 0$$

19. [Ans. B]

First approximation using second order R-K method

$$y_1 = y_0 + k$$

$$k = \frac{k_1 + k_2}{2}$$

$$k_1 = hf(x_0, y_0)$$

$$k_2 = hf(x_0 + h, y_0 + k_1)$$

$$\text{Where } f(x, y) = \frac{dy}{dx}$$

In the above question $y_0 = 2, x_0 = 1, h = 0.2$

$$k_1 = 0.2 \left(3 - \frac{2}{1} \right) = 0.2$$

$$k_2 = 0.2 \left(3 - \frac{(2 + 0.2)}{1 + 0.2} \right) = 0.2333$$

$$k = 0.2166$$

$$y_1 = 2 + 0.2166 = 2.2166$$

20. [Ans. B]

For bisection method

$$\frac{b - a}{2^n} \leq \text{accuracy}$$

$$\frac{2 - 1}{2^n} \leq 10^{-4}$$

$$10^4 \leq 2^n$$

$$n = 14$$

Option (C) is correct answer.

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