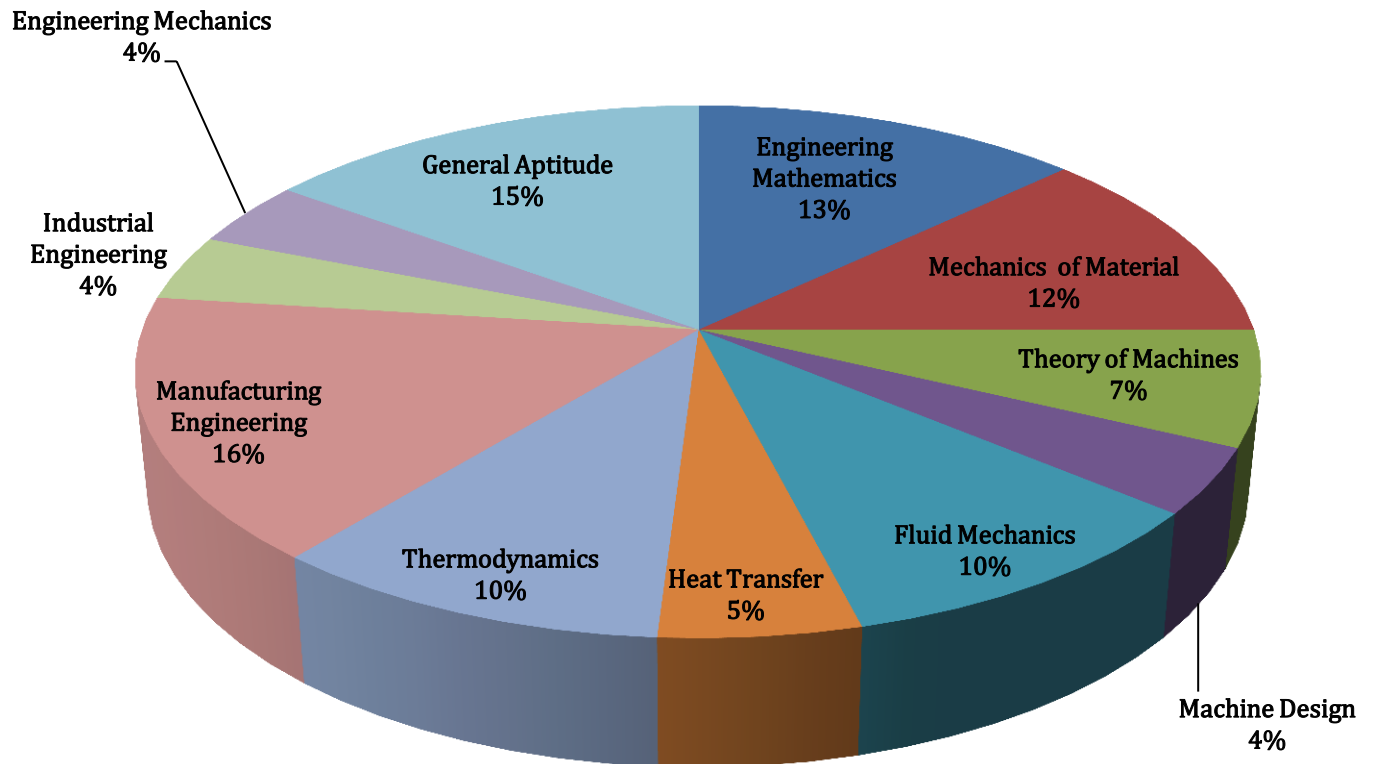


ANALYSIS OF GATE 2017*

Mechanical Engineering



ME ANALYSIS-2017_4-Feb_Morning

SUBJECT	Ques. No.	Topics Asked in Paper(Memory Based)	Level of Toughness	Total Marks
Engineering Mathematics	1 Marks:5 2 Marks:4	Calculus; Linear Algebra; Differential Equations; Probability and Distribution Numerical Methods	Moderate	13
Mechanics of Material	1 Marks: 4 2 Marks: 4	Torsion; Strain Energy Methods Simple Stress and Strain; Stresses in Beams Columns and Struts	Easy	12
Theory of Machines	1 Marks: 1 2 Marks: 3	Vibrations; Gear Trains	Easy	7
Machine Design	1 Marks: 2 2 Marks: 1	Design of Joints Design for Static Loading Design for Dynamic Loading	Easy, Moderate	4
Fluid Mechanics	1 Marks: 4 2 Marks: 3	Flow through Pipes; Fluid Kinematics Hydraulic Machines; Boundary Layer	Easy, Moderate	10
Heat Transfer	1 Marks: 1 2 Marks: 2	Heat Exchanger; Radiation; Conduction	Easy, Moderate	5
Thermodynamics	1 Marks: 2 2 Marks: 4	Properties of Gases and Pure Substances Entropy, Availability and Irreversibility Refrigeration	Moderate	10
Manufacturing Engineering	1 Marks: 4 2 Marks: 6	Machining and Machine Tools Operations Joining Process; Casting; Forming Process	Easy, Moderate	16
Industrial Engineering	1 Marks: 0 2 Marks: 2	Production, Planning and Control Operations Research	Moderate	4
Engineering Mechanics	1 Marks:2 2 Marks:1	Kinematics and Dynamics of Particle Kinematics and Dynamics of Rigid Body	Tough	4
General Aptitude	1 Marks: 5 2 Marks: 5	Numerical Ability; Verbal Ability	Tough	15
Total	65			100
Faculty Feedback	Majority of the question were direct concept based. MPE, TD, MOM, Maths and FM weightage was comparatively high. Surprise was GA, it was comparatively tough as compared to the last year.			

GATE 2017 Examination Mechanical Engineering

Test Date: 04/02/2017
Test Time: 9:00 AM to 12:00 PM
Subject Name: MECHANICAL ENGINEERING

Section: General Aptitude

1. P, Q and R talk about S's car collection. P states that S has at least 3 cars. Q believes that S has less than 3 cars. R indicates that to his knowledge, S has at least one car. Only one of P, Q and R is right. The number of cars owned by S is
- (A) 0
(B) 1
(C) 3
(D) Cannot be determined

[Ans. A]

P states that S has at least 3 cars = ≥ 3

Q believes that S has less than 3 cars = < 3

R indicates that S has at least one car = ≥ 1

P's and Q's statements are exactly opposite in nature and R's statement is proportional to P's statement.

From the given data, only one person statement is right as it mean that two person statements are wrong. i.e., P and R when S has zero cars.

2. In a company with 100 employees, 45 earn ₹ 20,000 per month, 25 earn ₹ 30,000, 20 earn ₹ 40,000, 8 earn ₹ 60,000, and 2 earn ₹ 1,50,000. The median of the salaries is
- (A) ₹ 20,000
(B) ₹ 30,000
(C) ₹ 32,300
(D) ₹ 40,000

Among the group of 100 employees middle persons are 50th and 51st persons, their salaries are ₹ 30,000 each

$$\text{Median} = \frac{3000 + 3000}{2} = 3000$$

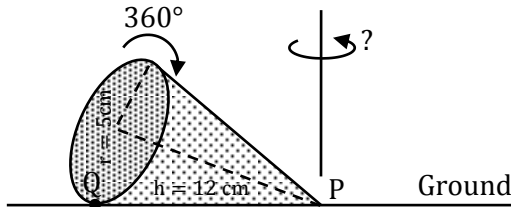
[Ans. B]

3. As the two speakers became increasingly agitated, the debate became _____.
- (A) lukewarm
(B) poetic
(C) forgiving
(D) heated

[Ans. D]

Strengthening sentence. One part of the sentence i.e., “Two speakers became increasingly agitated” strongly supports the latter part of the sentence “the debate became heated”

4. A right-angled cone (with base radius 5 cm and height 12 cm), as shown in the figure below, is rolled on the ground keeping the point P fixed until the point Q (at the base of the cone, as shown) touches the ground again.



By what angle (in radians) about P does the cone travel?

- (A) $\frac{5\pi}{2}$
 (B) $\frac{5\pi}{24}$
 (C) $\frac{24\pi}{5}$
 (D) $\frac{10\pi}{13}$

[Ans. D]

From the given data, base radius = 5 cm,

Height of the cone = 12 cm,

Slant height of the cone (l) = $\sqrt{(h^2) + r^2} = \sqrt{(12^2) + (5)^2} = \sqrt{144 + 25} = \sqrt{169} = 13\text{cm}$

It is rolled on the ground through the fixed point 'P'

We have relation $r = \frac{\theta}{2\pi} \times R$

Where, r = radius of the cone,

R = radius on the ground = slant height

θ = Angle about P does the cone travel

$$5 = \frac{\theta}{2\pi} \times 13 \Rightarrow \theta = \frac{10\pi}{13}$$

5. He was one of my best _____ and I felt his loss _____.
- (A) friend, keenly
 (B) friends, keen
 (C) friend, keener
 (D) friends, keenly

[Ans. D]

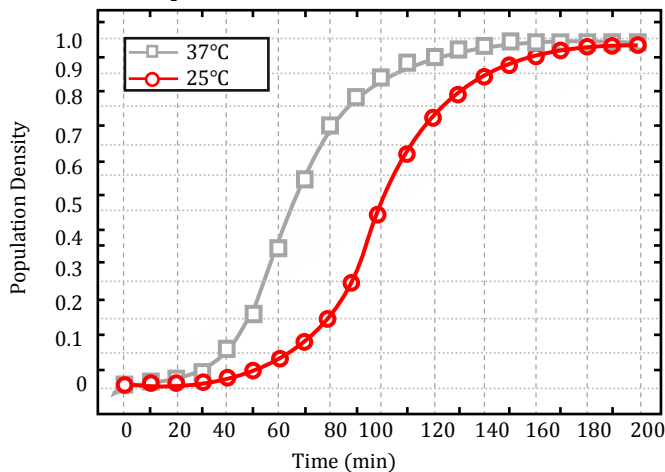
'One of' the expression always takes 'plural noun' is 'friends' and 'keenly' is an adverb modifies the verb felt. So the right option is 'D'.

6. Two very famous sportsmen Mark and Steve happened to be brothers, and played for country K. Mark teased James, an opponent from country E, "There is no way you are good enough to play for your country." James replied, "Maybe not, but at least I am the best player in my own family." Which one of the following can be inferred from this conversation?
- (A) Mark was known to play better than James
 (B) Steve was known to play better than Mark
 (C) James and Steve were good friends
 (D) James played better than Steve

[Ans. B]

When Mark said that 'James' was not good enough to play for his country and James acknowledged that he wasn't if infers that James played better than Steve.

7. The growth of bacteria (lactobacillus) in milk leads to curd formation. A minimum bacterial population density of 0.8 (in suitable units) is needed to form curd. In the graph below, the population density of lactobacillus in 1 litre of milk is plotted as a function of time, at two different temperatures, 25°C and 37°C



Consider the following statements based on the data shown above:

- i. The growth in bacterial population stops earlier at 37°C as compared to 25°C
 ii. The time taken for curd formation at 25°C is twice the time taken at 37°C

Which one of the following options is correct?

- (A) Only i
 (B) Only ii
 (C) Both i and ii
 (D) Neither i nor ii

[Ans. A]

From the graph statement (i) is correct

- the time taken for curd formation @ 25°C = 120 min
 - the time taken for curd formation @ 37°C = 80 min
- ∴ Statement (ii) is not correct.

8. Let S_1 be the plane figure consisting of the points (x,y) given by the inequalities $|x-1| \leq 2$ and $|y+2| \leq 3$. Let S_2 be the plane figure given by the inequalities $x-y \geq -2, y \geq 1$, and $x \leq 3$. Let S be the union of S_1 and S_2 . The area of S is
- (A) 26
(B) 28
(C) 32
(D) 34
- [Ans. C]**

9. "Here, throughout the early 1820s, Stuart continued to fight his losing battle to allow his sepoys to wear their caste-marks and their own choice of facial hair on parade, being again reprimanded by the commander-in-chief. His retort that 'A stronger instance than this of European prejudice with relation to this country has never come under my observations' had no effect on his superiors." According to this paragraph, which of the statements below is most accurate?
- (A) Stuart's commander-in-chief was moved by this demonstration of his prejudice.
(B) The Europeans were accommodating of the sepoys' desire to wear their caste-marks.
(C) Stuart's 'losing battle' refers to his inability to succeed in enabling sepoys to wear caste-marks.
(D) The commander-in-chief was exempt from the European prejudice that dictated how the sepoys were to dress.

[Ans. C]

The key word is 'losing battle'

10. What is the sum of the missing digits in the subtraction problem below?

$$\begin{array}{r} 5 \quad _ _ _ _ \\ -48_89 \\ \hline 1111 \end{array}$$

- (A) 8
(B) 10
(C) 11
(D) Cannot be determined

[Ans. D]

Section: Technical

1. For steady flow of a viscous incompressible fluid through a circular pipe of constant diameter, the average velocity in the fully developed region is constant. Which one of the following statements about the average velocity in the developing region is TRUE?
- (A) It increases until the flow is fully developed.
(B) It is constant and is equal to the average velocity in the fully developed region.
(C) It decreases until the flow is fully developed.
(D) It is constant but is always lower than the average velocity in the fully developed region.

[Ans. B]

Average velocity is defined as ratio of discharge and area. In fully developed region average velocity is constant hence discharge is constant. As long as discharge and area is constant average velocity in developing region as well has to be constant.

2. A motor driving a solid circular steel shaft transmits 40 kW of power at 500 rpm. If the diameter of the shaft is 40 mm, the maximum shear stress in the shaft is _____ MPa.

[Ans. *]Range: 60 to 61

Power transmitted, $P = 40 \text{ kW}$

Rotations, $N = 500 \text{ rpm}$

Diameter of shaft, $d = 40 \text{ mm}$

Power transmission

$$P = \frac{2\pi NT}{60}$$

$$40 = \frac{2\pi \times 500 \times T}{60}$$

$$T = 0.76 \text{ kN-m}$$

$$\text{Maximum shear stress developed, } \tau_{\max} = \frac{16T}{\pi d^3}$$

$$\tau_{\max} = \frac{16(0.76 \times 10^6)}{\pi \times 40^3} = 60.47 \text{ MPa}$$

3. Consider the two-dimensional velocity field given by $\vec{V} = (5 + a_1x + b_1y)\hat{i} + (4 + a_2x + b_2y)\hat{j}$, where a_1, b_1, a_2 and b_2 are constants. Which one of the following conditions needs to be satisfied for the flow to be incompressible?

(A) $a_1 + b_1 = 0$

(B) $a_1 + b_2 = 0$

(C) $a_2 + b_2 = 0$

(D) $a_2 + b_1 = 0$

[Ans. B]

For incompressibility $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial}{\partial x}(5 + a_1x + b_1y) + \frac{\partial}{\partial y}(4 + a_2x + b_2y) = 0$$

$$\text{or } a_1 + b_2 = 0$$

4. The Poisson's ratio for a perfectly incompressible linear elastic material is

(A) 1

(B) 0.5

(C) 0

(D) Infinity

[Ans. B]

Poisson's ratio of perfectly incompressible (non-dilatant) material is 0.5 (maximum).

5. Metric thread of 0.8 mm pitch is to be cut on a lathe. Pitch of the lead screw is 1.5 mm. If the spindle rotates at 1500 rpm, the speed of rotation of the lead screw (rpm) will be _____

[Ans. *] Range: 800 to 800

$$P_{\text{Job}} = 0.8\text{mm} \quad P_{\text{LS}} = 1.5\text{mm}$$

$$N_{\text{Job}} = 1500\text{rpm} \quad N_{\text{LS}} = ?$$

$$G. R = T. V = \frac{N_f}{N_d} = \frac{N_{\text{LS}}}{N_{\text{Job}}} = \frac{P_{\text{Job}}}{P_{\text{LS}}} = \frac{0.8}{1.5}$$

$$N_{\text{LS}} = \frac{0.8}{1.5} \times 1500 = 800 \text{ rpm}$$

6. In the engineering stress-strain curve for mild steel, the Ultimate Tensile Strength (UTS) refers to
- (A) Yield stress
(B) Proportional limit
(C) Maximum stress
(D) Fracture stress

[Ans. C]

7. The damping ratio for a viscously damped spring mass system, governed by the relationship

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F(t), \text{ is given by}$$

(A) $\sqrt{\frac{c}{mk}}$

(B) $\frac{c}{2\sqrt{km}}$

(C) $\frac{c}{\sqrt{km}}$

(D) $\sqrt{\frac{c}{2mk}}$

[Ans. B]

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

$$\text{We have } \frac{c}{m} = \frac{2c}{2\sqrt{km}} \omega_n = 2 \frac{c}{c_c} \omega_n$$

$$\text{Damping ratio} = \frac{c}{c_c} = \frac{c}{2\sqrt{km}}$$

8. In an arc welding process, welding speed is doubled. Assuming all other process parameters to be constant, the cross sectional area of the weld bead will
- (A) increase by 20%
(B) increase by 50%
(C) reduce by 25%
(D) reduce by 50%

[Ans. D]

$$V_2 = 2V_1$$

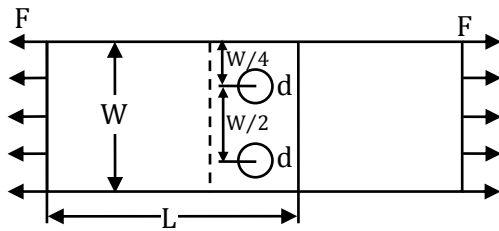
$$A_{w1} \times V_1 = A_{w2} \times V_2$$

$$A_{w2} = \frac{A_{w1} \times V_1}{V_2} = \frac{A_{w1} \times V_1}{2V_1} = \frac{A_{w1}}{2}$$

$$\% \text{ change} = \frac{A_{w2} - A_{w1}}{A_{w1}} = \frac{0.5 - 1}{1} = -0.5$$

-50% → reduced by 50%

9. Consider the schematic of a riveted lap joint subjected to tensile load F , as shown below. Let d be the diameter of the rivets, and S_f be the maximum permissible tensile stress in the plates. What should be the minimum value for the thickness of the plates to guard against tensile failure of the plates? Assume the plates to be identical.



- (A) $\frac{F}{S_f(W - 2d)}$
 (B) $\frac{F}{S_f W}$
 (C) $\frac{F}{S_f(W - d)}$
 (D) $\frac{2F}{S_f W}$

[Ans. A]

s_f = permissible tensile stress in plates

d = diameter of rivet

F = tensile load

t = ?

For tearing failure of plates,

$$F = (W - 2d)t \times \frac{S_{yt}}{FS} = (W - 2d) \times t \times S_f$$

$$\therefore t = \frac{F}{(W - 2d) \times s_f}$$

10. The value of

$$\lim_{x \rightarrow 0} \frac{x^3 - \sin(x)}{x} \text{ is}$$

- (A) 0
 (B) 3
 (C) 1
 (D) -1

[Ans. D]

$$\lim_{x \rightarrow 0} \left(\frac{x^3 - \sin x}{x} \right) \left(\frac{0}{0} \text{ form} \right)$$

Applying L' Hospital's Rule

$$= \lim_{x \rightarrow 0} (3x^2 - \cos x)$$

$$= -\cos 0 = -1$$

11. In a metal forming operation when the material has just started yielding, the principal stresses are $\sigma_1 = +180$ MPa, $\sigma_2 = -100$ MPa, $\sigma_3 = 0$. Following Von Mises' criterion, the yield stress is _____ MPa.

[Ans. *]Range: 245 to 246

$$\sigma_1 = 180 \text{ MPa}, \sigma_2 = -100 \text{ MPa}, \sigma_3 = 0$$

Von Mises yield stress,

$$\begin{aligned} \frac{\sigma_{yt}}{F_s} &= \sqrt{\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2} \\ &= \sqrt{180^2 + 100^2 - 180 \times (-100)} = 245.76 \text{ Mpa} \end{aligned}$$

12. The product of eigenvalues of the matrix P is

$$P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

- (A) -6
(B) 2
(C) 6
(D) -2

[Ans. B]

$$\text{Given } P = \begin{bmatrix} 2 & 0 & 1 \\ 4 & -3 & 3 \\ 0 & 2 & -1 \end{bmatrix}$$

$$\text{Product of eigen values} = \det(A) = 2(3 - 6) + 8 = 2$$

13. The molar specific heat at constant volume of an ideal gas is equal to 2.5 times the universal gas constant (8.314 J/mol. K). When the temperature increases by 100 K, the change in molar specific enthalpy is _____ J/mol.

[Ans. *]Range: 2908 to 2911

$$C_v = 2.5R$$

$$R = 8.314 \text{ J/mol.k}$$

$$dT = 100 \text{ k}$$

$$C_p = C_v + R = 2.5R + R = 3.5R$$

$$\begin{aligned} dh &= C_p dT = 3.5 \times 8.314 \times 100 \\ &= 2909.9 \text{ J/mol} \end{aligned}$$

14. Consider the following partial differential equation for $u(x, y)$ with the constant $c > 1$:

$$\frac{\partial u}{\partial y} + c \frac{\partial u}{\partial x} = 0$$

Solution of this equation is

(A) $u(x, y) = f(x + cy)$

(B) $u(x, y) = f(x - cy)$

(C) $u(x, y) = f(cx + y)$

(D) $u(x, y) = f(cx - y)$

[Ans. B]

$$\frac{\partial u}{\partial y} + C \frac{\partial u}{\partial x} = 0$$

Method-I

Verify the options.

Let us consider option (B)

$$u = f(x - cy)$$

$$\frac{\partial u}{\partial y} = -Cf'(x - Cy), \frac{\partial u}{\partial x} = f'(x - Cy)$$

$$\frac{\partial u}{\partial x} = C \frac{\partial u}{\partial y} = 0 \text{ (satisfied)}$$

Option (B) is correct.

Method-II

$$\text{Given } \frac{\partial u}{\partial y} + C \frac{\partial u}{\partial x} = 0$$

The Lagrange's auxiliary equation

$$\frac{dx}{c} = \frac{dy}{1} = \frac{dz}{0}$$

Taking first two fractions

$$\frac{dx}{c} = \frac{dy}{1} \Rightarrow dx = C dy$$

$$\Rightarrow \int dx = \int C dy$$

$$\Rightarrow x = Cy + k_1$$

$$\Rightarrow x - Cy = k_1 \dots \dots \textcircled{1}$$

Taking last two fractions

$$\frac{dy}{1} = \frac{dz}{0}$$

$$\Rightarrow dz = 0$$

$$\Rightarrow \int dz = \int 0$$

$$z = k_2 \dots \dots \textcircled{2}$$

∴ The solution is $\phi(x - Cy, z) = 0$

or $z = \phi(x - Cy)$

15. Water (density = 1000 kg/m³) at ambient temperature flows through a horizontal pipe of uniform cross section at the rate of 1 kg/s. If the pressure drop across the pipe is 100 kPa, the minimum power required to pump the water across the pipe, in Watts, is _____

[Ans. *]Range: 100 to 100

$$\text{Power} = \dot{m}vdp$$

$$\rho = 1000 \text{ kg/m}^3$$

$$v = \frac{1}{\rho} = \frac{1}{1000} \text{ m}^3/\text{kg}$$

$$dP = 100 \text{ kPa} = 10^5 \text{ Pa}$$

$$\dot{m} = 1 \text{ kg/sec}$$

$$\text{Power} = 1 \times \frac{1}{1000} \times 10^5 = 100 \text{ W}$$

16. Consider a beam with circular cross-section of diameter d . The ratio of the second moment of area about the neutral axis to the section modulus of the area is

(A) $\frac{d}{2}$

(B) $\frac{\pi d}{2}$

(C) d

(D) πd

[Ans. A]

$$\frac{I_{NA}}{\left(\frac{I_{NA}}{y_{\max}}\right)} = y_{\max} = \frac{d}{2}$$

17. A Saturated steam at 100°C condenses on the outside of a tube. Cold fluid enters the tube at 20°C and exits at 50°C . The value of the Log Mean Temperature Difference (LMTD) is _____ $^\circ\text{C}$

[Ans. *]Range: 63.5 to 64

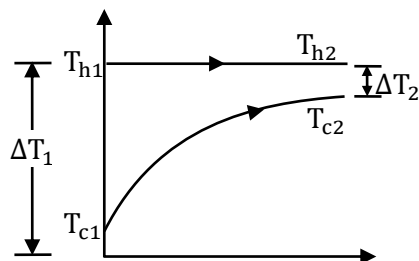
Hot fluid:

$$T_{h1} = T_{h2} = 100^\circ\text{C}$$

Cold fluid

$$T_{c1} = 20^\circ\text{C}$$

$$T_{c2} = 50^\circ\text{C}$$



$$\Delta T_1 = T_{h1} - T_{c1} = 100 - 20 = 80^\circ\text{C}$$

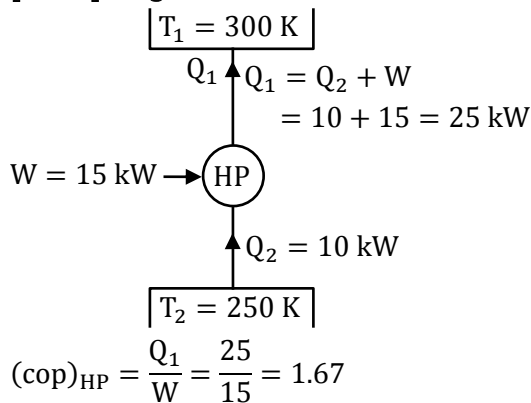
$$\Delta T_2 = T_{h2} - T_{c2} = 100 - 50 = 50^\circ\text{C}$$

$$\text{LMTD} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{80 - 50}{\ln\left(\frac{80}{50}\right)}$$

$$\text{LMTD} = 63.82^\circ\text{C}$$

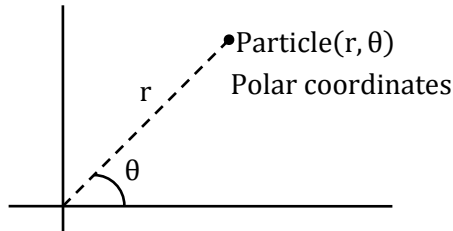
18. A heat pump absorbs 10 kW of heat from outside environment at 250 K while absorbing 15 kW of work. It delivers the heat to a room that must be kept warm at 300 K. The Coefficient of performance (COP) of the heat pump is _____

[Ans. *]Range: 1.60 to 1.70



19. A Particle of unit mass is moving on a plane. Its trajectory, in polar coordinates, is given by $r(t) = t^2$, $\theta(t) = t$, where t is time. The kinetic energy of the particle at time $t = 2$ is
- (A) 4
(B) 12
(C) 16
(D) 24

[Ans. C]



$$r(t = 2\text{sec}) = t^2; V = \frac{dr}{dt} = 2t; V(t = 2) = 2 \times 2 = 4 \text{ m/s}; r = 2 \times 2 = 4 \text{ m}$$

$$\theta(t) = t; \omega = \frac{d\theta}{dt} = 1 \text{ rad/s}; I = mr^2 = 1 \times 4^2 = 16 \text{ kg-m}^2$$

$$\begin{aligned} \text{Total kinetic energy} &= \frac{1}{2} mV^2 + \frac{1}{2} I\omega^2 \\ &= \frac{1}{2} \times 1 \times r^2 + \frac{1}{2} \times 16 \times 1^2 \\ &= 8 + 8 = 16 \text{ N-m} \end{aligned}$$

20. Cylindrical pins of a diameter $15^{\pm 0.020}$ mm are being produced on a machine. Statistical quality control tests show a mean of 14.995 mm and standard deviation of 0.004 mm. The process capability index C_p is
- (A) 0.833
(B) 1.667
(C) 3.333
(D) 3.750

[Ans. B]

$$\text{USL} = 15.02 \text{ mm}, \text{LSL} = 14.98 \text{ mm}, \sigma = 0.004 \text{ mm}$$

$$\text{Process capability ratio} = \frac{USL - LSL}{6\sigma} = \frac{15.02 - 14.98}{6 \times 0.004} = 1.667$$

CPU = Upper specification calculation

CPL = Lower specification calculation

$$\mu = 14.995 \text{ mm}$$

$$(\text{CPU}) = \frac{USL - \mu}{3\sigma} = \frac{15.02 - 14.995}{3 \times 0.004} = 2.083$$

$$(\text{CPL}) = \frac{\mu - LSL}{3\sigma} = \frac{14.995 - 14.98}{3 \times 0.004} = 1.25$$

Process capability index = minimum (CPU, CPL) = 1.25

Here, process capability ratio is 1.667. There is no option matching. Out of given option best option (B).

21. Match the processes with their characteristics

Process	Characteristics
P. Electrical Discharge Machining	1. No residual stress
Q. Ultrasonic Machining	2. Machining of electrically conductive materials
R. Chemical Machining	3. Machining of glass
S. Ion Beam Machining	4. Nano-machining

(A) P-2, Q-3, R-1, S-4

(B) P-3, Q-2, R-1, S-4

(C) P-3, Q-2, R-4, S-1

(D) P-2, Q-4, R-3, S-1

[Ans. A]

USM is used only for machining of highly brittle materials such as glass.

22. A six-face fair dice is rolled a large number of times. The mean value of the outcomes is _____

[Ans. *]Range: 3.5 to 3.5

Let, x = outcomes of the die

The probability distribution of X is

X_0	1	2	3	4	5	6
$P(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

$$E(x) = \sum_{x=1}^6 X_0 P(x)$$

$$= 1 \times \frac{1}{6} + 2 \times \frac{1}{6} + 3 \times \frac{1}{6} + 4 \times \frac{1}{6} + 5 \times \frac{1}{6} + 6 \times \frac{1}{6}$$

$$= 3.5$$

23. The differential equation $\frac{d^2y}{dx^2} + 16y = 0$ for $y(x)$ with the two boundary conditions

$$\left. \frac{dy}{dx} \right|_{x=0} = 1 \text{ and } \left. \frac{dy}{dx} \right|_{x=\frac{\pi}{2}} = -1 \text{ has}$$

(A) no solution

(B) exactly two solutions

- (C) exactly one solution
- (D) infinitely many solutions

[Ans. A]

Given $\frac{d^2y}{dx^2} + 16y = 0$

The auxiliary equation is $D^2 + 16 = 0$

$D = \pm 4i$

The general solution is

$y = C_1 \cos 4x + C_2 \sin 4x \dots \dots \dots \textcircled{1}$

$\frac{dy}{dx} = -4C_1 \sin 4x + 4C_2 \cos 4x \dots \dots \dots \textcircled{2}$

At $x = 0, \frac{dy}{dx} = 1, \textcircled{2} \Rightarrow 1 = 4C_2$

$\Rightarrow C_2 = \frac{1}{4}$

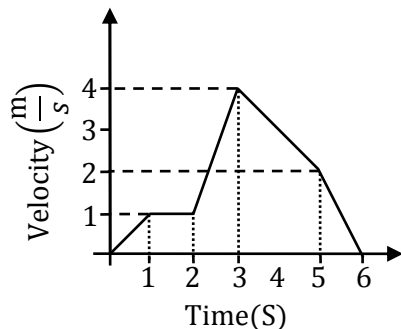
At $x = \frac{\pi}{2}, \frac{dy}{dx} = -1, \textcircled{2} \Rightarrow -1 = -4C_1 \sin 2\pi + 4C_2 \cos 2\pi$

$\Rightarrow 4C_2 = -1$

$\Rightarrow C_2 = -\frac{1}{4}$

\therefore It has no solution

24. The following figure shows the velocity-time plot for a particle travelling along a straight line. The distance covered by the particle from $t = 0$ to $t = 5$ s is _____ m.



[Ans. *]Range: 10 to 10

Total distance covered = Area under velocity – time graph

$= \frac{1}{2} \times 1 \times 1 + 1 \times 1 + 1 \times 1 + \frac{1}{2} \times 3 \times 1 + 2 \times 2 + \frac{1}{2} \times 2 \times 2 = 10 \text{ m}$

25. Which one of the following is NOT a rotating machine?
- (A) Centrifugal pump
 - (B) Gear pump
 - (C) Jet pump
 - (D) Vane pump

[Ans. C]

Pressure rise is produced due to mixing with high speed jet.

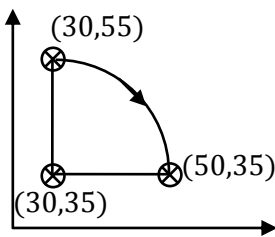
26. A Circular arc on a part profile is being machined on a vertical CNC milling machine. CNC part program using metric units with absolute dimensions is listed below:

```
-----
N60 G01 X 30 Y 55 Z -5 F50
N70 G02 X 50 Y 35 R 20
N80 G01 Z 5
-----
```

The coordinates of the centre of the circular arc are:

- (A) (30, 55)
- (B) (50, 55)
- (C) (50, 35)
- (D) (30, 35)

[Ans. D]



27. P (0, 3), Q (0.5, 4), and R (1, 5) are three points on the curve defined by $f(x)$. Numerical integration is carried out using both Trapezoidal rule and Simpson's rule within limits $x = 0$ and $x = 1$ for the curve. The difference between the two results will be

- (A) 0
- (B) 0.25
- (C) 0.5
- (D) 1

[Ans. A]

x	0	0.5	1
y	3	4	5

Where, $h = \frac{1}{2} = 0.5$

Using Trapezoidal rule,

$$\begin{aligned} \int_0^1 f(x)dx &= \frac{h}{2} [(y_0 + y_2) + 2(y_1)] \\ &= \frac{1}{4} [(3 + 5) + 2(4)] \\ &= 4 \end{aligned}$$

Using Simpson's rule

$$\begin{aligned} \int_0^1 f(x)dx &= \frac{h}{3} [(y_0 + y_2) + 4y_1] \\ &= \frac{1}{6} [8 + 16] \\ &= 4 \end{aligned}$$

\therefore The difference between two results = 0

28. For a steady flow, the velocity field is $\vec{V} = (-x^2 + 3y)\vec{i} + (2xy)\vec{j}$. The magnitude of the acceleration of a particle at (1, -1) is

- (A) 2
- (B) 1
- (C) $2\sqrt{5}$
- (D) 0

[Ans. C]

$$u = -x^2 + 3y = -1^2 + 3(-1) = -4$$

$$v = 2xy = 2(1)(-1) = -2$$

$$\frac{\partial u}{\partial x} = -2x = -2, \quad \frac{\partial u}{\partial y} = 3, \quad \frac{\partial v}{\partial x} = 2y = 2(-1) = -2$$

$$\frac{\partial v}{\partial y} = 2x = 2(1) = 2$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = (-4)(-2) + (-2) \times 3 = 2$$

$$a_y = u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = (-4)(-2) + (-2)(2) = 4$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(2^2 + 4^2)} = \sqrt{20} = 2\sqrt{5}$$

29. Following data refers to the jobs (P, Q, R, S) which have arrived at a machine for scheduling. The shortest possible average flow time is _____ days

Job	Processing Time (days)
P	15
Q	9
R	22
S	12

[Ans. *]Range: 31 to 31

SPT rule gives shortest average flow time

Job	Processing Time (T_i)	Completion time (C_i)
Q	9	9
S	12	21
P	15	36
R	22	58
		$\Sigma C_i = 124$

$$\text{Mean flow time} = \frac{\sum c_i}{n} = \frac{124}{4} = 31$$

30. Consider the matrix $P = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \\ 0 & 1 & 0 \\ -\frac{1}{\sqrt{2}} & 0 & \frac{1}{\sqrt{2}} \end{bmatrix}$

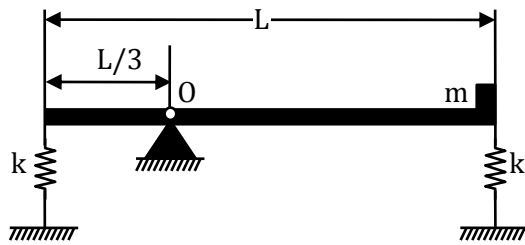
Which one of the following statements about P is INCORRECT?

- (A) Determinant of P is equal to 1
- (B) P is orthogonal
- (C) Inverse of P is equal to its transpose
- (D) All eigenvalues of P are real numbers

[Ans. D]

The eigen values of an orthogonal matrix are of magnitude 1 and are real (A) complex conjugate pairs

31. A thin uniform rigid bar of length L and mass M is hinged at point O, located at a distance of $\frac{L}{3}$ from one of its ends. The bar is further supported using springs, each of stiffness k, located at the two ends. A particle of mass $m = \frac{M}{4}$ is fixed at one end of the bar, as shown in the figure. For small rotations of the bar about O, the natural frequency of the system is



- (A) $\sqrt{\frac{5k}{M}}$
- (B) $\sqrt{\frac{5k}{2M}}$
- (C) $\sqrt{\frac{3k}{2M}}$
- (D) $\sqrt{\frac{3k}{M}}$

[Ans. B]

Take the moment about the hinge 'O'

The equation of motion is

$$\left(I_0 + m \left(\frac{2L}{3} \right)^2 \right) \ddot{\theta} + \left(k \left(\frac{L}{3} \right)^2 + k \left(\frac{2L}{3} \right)^2 \right) \theta = 0$$

$$I_0 = \frac{ML^2}{12} + M \left(\frac{L}{6} \right)^2 = \frac{ML^2}{9}, m = \frac{M}{4}$$

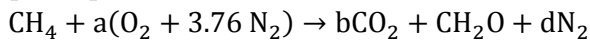
$$\left(\frac{ML^2}{9} + \frac{4ML^2}{9 \times 4} \right) \ddot{\theta} + \frac{5kL^2}{9} \theta = 0$$

$$(2ML^2)\ddot{\theta} + \frac{5kL^2}{9}\theta = 0$$

$$\text{Natural frequency, } \omega_n = \sqrt{\frac{k_{eq}}{m_{eq}}} = \sqrt{\frac{\left(\frac{5kL^2}{9}\right)}{2ML^2}} = \sqrt{\frac{5k}{2M}}$$

32. Air contains 79% N₂ and 21% O₂ on a molar basis. Methane (CH₄) is burned with 50% excess air than required stoichiometrically. Assuming complete combustion of methane, the molar percentage of N₂ in the products is _____

[Ans. *]Range: 73 to 74



$$\text{C: } b=1$$

$$\text{H: } 2c=4$$

$$c = 2$$

$$\text{O: } 2a = 2b + c$$

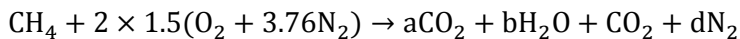
$$a = b + c/2$$

$$= 1 + \frac{2}{2} = 2$$

$$\text{N}_2 = 7.52a = 2d$$

$$d = 3.76a = 3.76 \times 2 = 7.52$$

50% excess air is used



$$\text{C: } a = 1$$

$$\text{H: } 2b = 4$$

$$b = 2$$

$$\text{O: } 2a + b + 2c = 6$$

$$2 \times 1 + 2 + 2c = 6$$

$$2c = 2$$

$$c = 1$$

$$\text{N: } 2d = 6 \times 3.76$$

$$d = 3 \times 3.76 = 11.28$$

$$\text{CO}_2 = 1 \text{ mole}$$

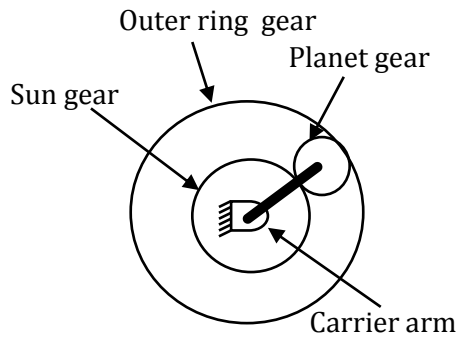
$$\text{H}_2\text{O} = 2 \text{ moles}$$

$$\text{O}_2 = 1 \text{ mole}$$

$$\text{N}_2 = 11.28 \text{ moles}$$

$$\begin{aligned} \text{Percentage nitrogen in products} &= \frac{\text{Number of moles of N}_2}{\text{Total number of moles}} \times 100 \\ &= \left(\frac{11.28}{1 + 2 + 1 + 11.28} \right) \times 100 = \frac{11.28}{15.28} \times 100 = 73.82\% \end{aligned}$$

33. In an epicyclic gear train, shown in the figure, the outer ring gear is fixed, while the sun gear rotates counterclockwise at 100 rpm. Let the number of teeth on the sun, planet and outer gears to be 50, 25 and 100 respectively. The ratio of magnitudes of angular velocity of the planet gear to the angular velocity of the carrier arm is _____.



[Ans. *]Range: 3 to 3

$$N_R = 0$$

$$N_S = 100 \text{ rpm (ccw)}$$

$$\frac{N_P}{N_a} = ?$$

$$\frac{N_S - N_a}{N_R - N_a} = -\frac{T_R}{T_S}$$

$$\frac{100 - N_a}{0 - N_a} = -\frac{100}{50}$$

$$\frac{100 - N_a}{-N_a} = -2$$

$$N_a = +\frac{100}{3} \text{ rpm(ccw)}$$

$$\frac{N_S - N_a}{N_P - N_a} = -\frac{T_P}{T_S}$$

$$\frac{100 - N_a}{N_P - N_a} = -\frac{25}{50}$$

$$\frac{100 - N_a}{N_P - N_a} = -\frac{1}{2}$$

$$\therefore N_P = 3 N_a - 200 = 3 \times \frac{100}{3} - 200 = -100 \text{ rpm} = 100 \text{ rpm(cw)}$$

$$\left| \frac{N_P}{N_a} \right| = 3$$

34. Two cutting tools with tool life equations given below are being compared:

$$\text{Tool 1: } VT^{0.1} = 150$$

$$\text{Tool 2: } VT^{0.3} = 300$$

Where V is cutting speed in m/minute and T is tool life in minutes. The breakeven cutting speed beyond which Tool 2 will have a higher tool life is _____ m/minute.

[Ans. *]Range: 105 to 107

$$VT^{0.1} = 150 \Rightarrow T = \left(\frac{150}{V}\right)^{\frac{1}{0.1}} = \left(\frac{150}{V}\right)^{10}$$

$$VT^{0.3} = 300 \Rightarrow T = \left(\frac{300}{V}\right)^{\frac{1}{0.3}} = \left(\frac{300}{V}\right)^{\frac{10}{3}}$$

At BEP, T = T and V = V

$$T = T$$

$$\left(\frac{150}{V}\right)^{10} = \left(\frac{300}{V}\right)^{\frac{10}{3}}$$

$$\frac{V^{10}}{V^{\frac{10}{3}}} = V^{(10 - \frac{10}{3})} = \frac{(150)^{10}}{(300)^{\frac{10}{3}}}$$

$$V = 106.069$$

35. One kg of an ideal gas (gas constant, $R = 400 \text{ J/kg} \cdot \text{K}$; specific heat at constant volume, $c_v = 1000 \text{ J/kg} \cdot \text{K}$) at 1 bar, and 300 K is contained in a sealed rigid cylinder. During an adiabatic process, 100 kJ of work is done on the system by a stirrer. The increase in entropy of the system is _____ J/K.

[Ans. *]Range: 286 to 288

$$m = 1 \text{ kg}$$

$$R = 400 \text{ J/kg} \cdot \text{K}$$

$$C_v = 1000 \text{ J/kg} \cdot \text{K}$$

$$P = 100 \text{ kPa}$$

$$T_1 = 300 \text{ K}$$

$$W_{\text{stirrer}} = W_s = -100 \times 1000 \text{ J}$$

$$\text{Adiabatic process } dQ = 0$$

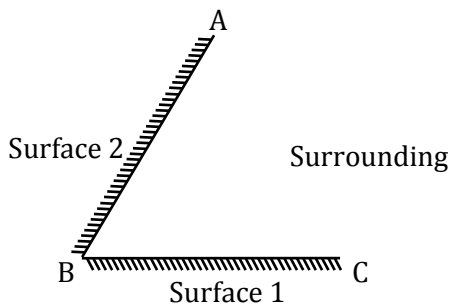
$$dQ - dW = dU = mC_v(T_2 - T_1)$$

$$0 - (-10^5) = 1 \times 1000(T_2 - 300)$$

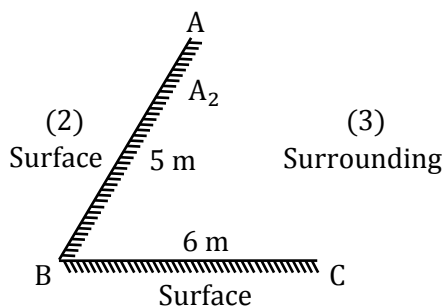
$$T_2 = 100 + 300 = 400 \text{ K}$$

$$d_s = mC_v \ln \frac{T_2}{T_1} = 1 \times 1000 \ln \frac{400}{300} = 287.68 \text{ J/K}$$

36. Two black surfaces, AB and BC, of lengths 5 m and 6 m, respectively, are oriented as shown. Both surfaces extend infinitely into the third dimension. Given that view factor $F_{12} = 0.5$, $T_1 = 800 \text{ K}$, $T_2 = 600 \text{ K}$, $T_{\text{surrounding}} = 300 \text{ K}$ and Stefan Boltzmann constant, $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\text{K}^4)$, the heat transfer rate from surface 2 to the surrounding environment is _____ kW.



[Ans. *]Range: 13.7 to 13.9



$$F_{12} = 0.5$$

$$T_1 = 800 \text{ K}$$

$$T_2 = 600 \text{ K}$$

$$T_{\text{surrounding}} = 300 \text{ K}$$

Let (3) denote the surrounding environment

(Assume unit width perpendicular to plane of figure)

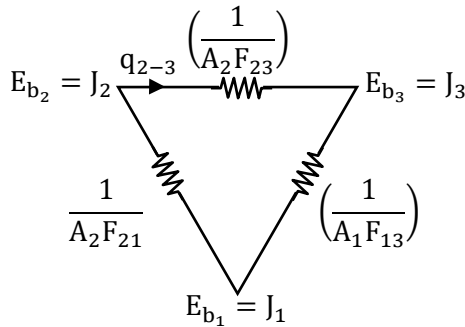
From summation rule.

$$F_{21} + F_{22}^0 + F_{23} = 1$$

$$A_1 F_{12} = A_2 F_{21} \Rightarrow 6 \times 1 \times 0.5 = (5 \times 1) \times F_{21}$$

$$F_{21} = \frac{6 \times 0.5}{5} = 0.6$$

$$\therefore F_{23} = 1 - F_{21} = 0.4$$



Since surrounding environment being large,

No surface has a surface resistance

$$\text{Heat transfer rate from (2) and (3)} = \frac{E_{b_2} - E_{b_3}}{\frac{1}{A_2 F_{23}}}$$

$$= \frac{\sigma(T_2^4 - T_3^4)}{\frac{1}{A_2 F_{23}}}$$

$$= (5.67 \times 10^{-8} (600^4 - 300^4)) \frac{1}{(5 \times 1 \times 0.4)} \text{ watt/metre} = 13.778 \text{ kW/metre}$$

Assume that heat exchange is per unit width perpendicular to plane of figure.

37. A 10 mm deep cylindrical cup with diameter of 15 mm is drawn from a circular blank. Neglecting the variation in the sheet thickness, the diameter (upto 2 decimal points accuracy) of the blank is _____ mm.

[Ans. *]Range: 28.71 to 28.73

$$h = 10 \text{ mm, } d = 15 \text{ mm,}$$

$$D = \sqrt{d^2 + 4dh} = \sqrt{15^2 + 4 \times 10 \times 15} = \sqrt{825} = 28.7228 \text{ mm}$$

38. For the vector $\vec{V} = 2yz\vec{i} + 3xz\vec{j} + 4xy\vec{k}$, the value of $\nabla \cdot (\nabla \times \vec{V})$ is

[Ans. *]Range: 0 to 0

$$\nabla \cdot (\nabla \times \vec{V}) = \text{div}(\text{curl } \vec{V}) = 0 [\text{vector Identity}]$$

39. The velocity profile inside the boundary layer for flow over a flat plate is given as $\frac{u}{U_\infty} = \sin\left(\frac{\pi y}{2\delta}\right)$, where U_∞ is the free stream velocity and ' δ ' is the local boundary layer thickness. If δ^* is the local displacement thickness, the value of $\frac{\delta^*}{\delta}$ is

- (A) $\frac{2}{\pi}$
 (B) $1 - \frac{2}{\pi}$
 (C) $1 + \frac{2}{\pi}$
 (D) 0

[Ans. B]

$$\begin{aligned}\delta^* &= \int_0^\delta \left(1 - \frac{u}{U_\infty}\right) dy = \int_0^\delta \left[1 - \sin\left(\frac{\pi y}{2\delta}\right)\right] dy \\ &= \left[y + \frac{2\delta}{\pi} \cos\left(\frac{\pi y}{2\delta}\right)\right]_0^\delta \\ &= \left[\delta + \frac{2\delta}{\pi} \cos\left(\frac{\pi}{2}\right) - 0 - \frac{2\delta}{\pi} \cos(0)\right] \\ &= \delta - \frac{2\delta}{\pi} \\ \therefore \frac{\delta^*}{\delta} &= 1 - \frac{2}{\pi}\end{aligned}$$

40. Heat is generated uniformly in a long solid cylindrical rod (diameter = 10 mm) at the rate of $4 \times 10^7 \text{ W/m}^3$. The thermal conductivity of the rod material is 25 W/m.K. Under steady state conditions, the temperature difference between the centre and the surface of the rod is ____ °C.

[Ans. *]Range: 10 to 10

Given data,

$$R = 5 \times 10^{-3} \text{ m}$$

$$q_g = 4 \times 10^7 \text{ W/m}^3$$

$$k = 25 \text{ W/mK}$$

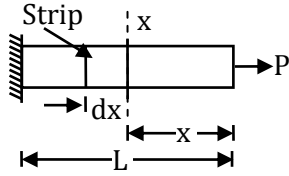
$$T_{\max} - T_s = \frac{q_g R^2}{4k}$$

$$T_{\max} - T_s = \frac{4 \times 10^7 \times (5 \times 10^{-3})^2}{4 \times 25}$$

$$T_{\max} - T_s = 10^\circ\text{C}$$

41. A horizontal bar, fixed at one end ($x = 0$), has a length of 1 m, and cross-sectional area of 100 mm^2 . Its elastic modulus varies along its length as given by $E(x) = 100 e^{-x} \text{ GPa}$, where x is the length coordinate (in m) along the axis of the bar. An axial tensile load of 10 kN is applied at the free end ($x = 1$). The axial displacement of the free end is ____ mm.

[Ans. *]Range: 1.70 to 1.72



Change in length of small strip,

$$\delta = \frac{P_{x-x}(dx)}{A_{x-x} E_{x-x}}$$

Total change in length of bar,

$$\delta_{\text{total}} = \int_0^L \delta = \int_0^L \frac{P_{x-x}(dx)}{A_{x-x} E_{x-x}}$$

$$P_{x-x} = P = \text{Constant}$$

$$A_{x-x} = A = \text{Constant}$$

$$E_{x-x} = 100 e^{-x} \text{ GPa}$$

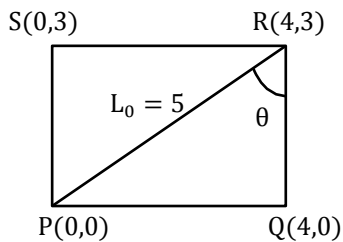
$$\delta_{\text{total}} = \frac{P}{A} \int_0^L \frac{dx}{100e^{-x}} = \frac{P}{10^9 \times 100A} \int_0^L \frac{dx}{e^{-x}}$$

$$= \frac{10 \times 10^3}{100 \times 10^{-6} \times 100 \times 10^9} (e^x)_0^L = 1.7182 \times 10^{-3}$$

$$\delta_{\text{total}} = 1.718 \text{ mm}$$

42. A rectangular region in a solid is in a state of plane strain. The (x,y) coordinates of the corners of the undeformed rectangle are given by $P(0,0)$, $Q(4,0)$, $R(4,3)$, $S(0,3)$. The rectangle is subjected to uniform strains, $\epsilon_{xx} = 0.001$, $\epsilon_{yy} = 0.002$, $\gamma_{xy} = 0.003$. The deformed length of the elongated diagonal, upto three decimal places, is ____ units.

[Ans. *]Range: 5.013 to 5.015



$$\epsilon_{xx} = 0.001, \epsilon_{yy} = 0.002, \gamma_{xy} = 0.003$$

$$\tan \theta = \frac{4}{3}, \quad \theta = 53.13$$

Normal strain along the oblique plane with inclination θ is

$$\epsilon_{\theta} = \left(\frac{\epsilon_{xx} + \epsilon_{yy}}{2} \right) + \left(\frac{\epsilon_{xx} - \epsilon_{yy}}{2} \right) \cos 2\theta + \frac{\gamma_{xy}}{2} \sin 2\theta$$

$$\epsilon_{\theta=53.13} = \left(\frac{0.001 + 0.002}{2} \right) + \left(\frac{0.001 - 0.002}{2} \right) \cos(2 \times 53.13) + \frac{0.003}{2} \sin(2 \times 53.13)$$

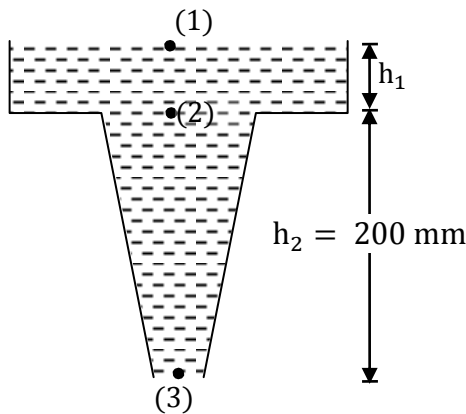
$$= 1.5 \times 10^{-3} + 1.399 \times 10^{-4} + 1.44 \times 10^{-3} = 0.003079$$

$$\epsilon_{\theta=53.13} = 0.003079 = \frac{\Delta L}{L_0} = \frac{L_1 - L_0}{L_0} = \frac{L_1 - 5}{5}$$

$$L_1 = 5 \times 0.003079 + 5 = 5.015 \text{ units}$$

43. A sprue in a sand mould has a top diameter of 20 mm and height of 200 mm. The velocity of the molten metal at the entry of the sprue is 0.5 m/s. Assume acceleration due to gravity as 9.8 m/s^2 and neglect all losses. If the mould is well ventilated, the velocity (upto 3 decimal points accuracy) of the molten metal at the bottom of the sprue is _____ m/s.

[Ans. *]Range: 2.04 to 2.07



$$d_2 = 20 \text{ mm,}$$

$$h_2 = 200 \text{ mm,}$$

$$V_2 = 0.5 \text{ m/s}$$

$$g = 9.8 \text{ m/s}^2$$

$$V_2 = \sqrt{2gh_1}$$

$$h_1 = \frac{V_2^2}{2g} = \frac{(0.5)^2}{2 \times 9.8} \times 1000$$

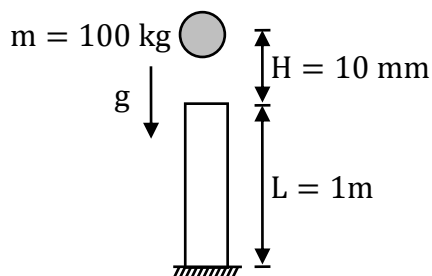
$$= 12.755 \text{ mm}$$

$$h_t = h_2 + h_1 = 200 + 12.755 = 212.755$$

$$V_3 = \sqrt{2gh_t} = \sqrt{2 \times 9.81 \times 1000 \times 212.755}$$

$$= 2042 \text{ mm/sec} = 2.042 \text{ m/sec}$$

44. A point mass of 100 kg is dropped onto a massless elastic bar (cross-sectional area = 100 mm^2 , length = 1 m, Young's modulus = 100 GPa) from a height H of 10 mm as shown (Figure is not to scale). If $g = 10 \text{ m/s}^2$, the maximum compression of the elastic bar is _____ mm.



[Ans. *]Range: 1.50 to 1.52

$$m = 100 \text{ kg ; } H = 10 \text{ mm ; } L = 1 \text{ m} = 1000 \text{ mm; } E = 100 \text{ GPa; } g = 10 \text{ m/sec}^2$$

$$W = 100 \times 10 = 1000 \text{ N}$$

$$\delta = \frac{WL}{AE} \left[1 + \sqrt{1 + \frac{2EAh}{WL}} \right]$$

Impact factor

$$= \frac{1000 \times 1000}{100 \times 100 \times 10^3} \left[1 + \sqrt{1 + \frac{2 \times 100 \times 10^3 \times 100 \times 10}{1000 \times 1000}} \right]$$

$$= \frac{1}{10} [1 + \sqrt{1 + 200}] = 1.5177 \text{ mm}$$

45. Moist air is treated as an ideal gas mixture of water vapour and dry air (molecular weight of air = 28.84 and molecular weight of water = 18). At a location, the total pressure is 100 kPa, the temperature is 30°C and the relative humidity is 55%. Given that the saturation pressure of water at 30°C is 4246 Pa, the mass of water vapour per kg of dry air is _____ grams.

[Ans. *]Range: 14.7 to 15.1

$$P_{\text{atm}} = 100 \text{ kPa}, T_{\text{sat}} = 30^\circ \text{C}, P_{\text{sat}} = 4.246 \text{ kPa}$$

$$\phi = 0.55$$

$$\phi = \frac{P_v}{P_{\text{sat}}}$$

$$P_v = \phi P_{\text{sat}} = 0.55 \times 4.246 = 2.3353 \text{ kPa}$$

$$\omega = 0.622 \times \frac{P_v}{P_{\text{atm}} - P_v} = 0.622 \times \frac{2.3353}{100 - 2.3353}$$

$$= 0.622 \times \frac{2.3353}{97.6647}$$

$$= 0.01487 \frac{\text{kg vap}}{\text{kg da}}$$

$$= 14.87 \text{ gm vap/kg. da}$$

46. Consider steady flow of an incompressible fluid through two long and straight pipes of diameters d_1 and d_2 arranged in series. Both pipes are of equal length and the flow is turbulent in both pipes. The friction factor for turbulent flow through pipes is of the form, $f = K(\text{Re})^{-n}$, where K and n are known positive constants and Re is the Reynolds number. Neglecting minor losses, the ratio of the frictional pressure drop in pipe 1 to that in pipe 2,

$\left(\frac{\Delta P_1}{\Delta P_2}\right)$, is given by

(A) $\left(\frac{d_2}{d_1}\right)^{5-n}$

(B) $\left(\frac{d_2}{d_1}\right)^5$

(C) $\left(\frac{d_2}{d_1}\right)^{3-n}$

(D) $\left(\frac{d_2}{d_1}\right)^{5+n}$

[Ans. A]

$$\Delta P = \rho g h_f = \rho g \times \frac{fLV^2}{2gd} = \frac{\rho fLV^2}{2d}$$

For both the pipes ρ & L are same

$$\therefore \Delta P \propto \frac{fV^2}{d}$$

$$\text{Now } V = \frac{Q}{A} \propto \frac{Q}{d^2}$$

$$f = kRe^{-n} = k \left(\frac{\rho Vd}{\mu} \right)^{-n} = k \left[\frac{\rho Qd}{\left(\frac{\pi}{4} d^2 \right) u} \right]^{-n}$$

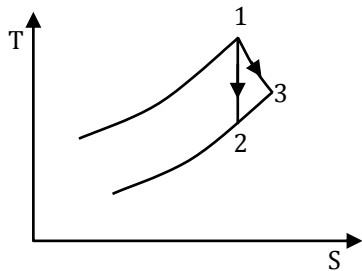
$$\propto \left(\frac{Q}{d} \right)^{-n}$$

$$\therefore \Delta P \propto \frac{\left(\frac{Q}{d} \right)^{-n} \times \left(\frac{Q}{d^2} \right)^2}{d} = \frac{Q^{2-n}}{d^{5-n}}$$

$$\therefore \frac{\Delta P_1}{\Delta P_2} = \frac{Q^{2-n}}{d_1^{5-n}} \times \frac{d_2^{5-n}}{Q^{2-n}} = \left(\frac{d_2}{d_1} \right)^{5-n}$$

47. The pressure ratio across a gas turbine (for air, specific heat at constant pressure, $c_p = 1040$ J/kg.K and ratio of specific heats, $\gamma = 1.4$) is 10. If the inlet temperature to the turbine is 1200 K and the isentropic efficiency is 0.9, the gas temperature at turbine exit is _____ K.

[Ans. *]Range: 675 to 684



$$C_p = 1040 \text{ J/kgK}$$

$$r = \frac{C_p}{C_v} = 1.4$$

$$r_p = \frac{P_2}{P_1} = 10$$

$$T_{\text{inlet}} = T_1 = 1200 \text{ K}$$

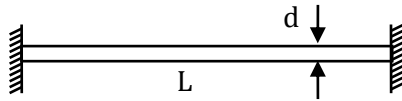
$$T_2 = \frac{T_1}{(r_p)^{\frac{\gamma-1}{\gamma}}} = \frac{1200}{10^{\frac{1.4-1}{1.4}}} = \frac{1200}{1.9307}$$

$$\text{Temperature at exit isentropic} = T_2 = 621.53 \text{ K}$$

$$\eta_T = 0.9 = \frac{T_1 - T_3}{T_1 - T_2}$$

$$\begin{aligned} T_3 &= T_1 - \eta_T(T_1 - T_2) \\ &= 1200 - 0.9(1200 - 621.53) \\ &= 679.38 \text{ K} = 680 \text{ K} \end{aligned}$$

48. An initially stress-free massless elastic beam of length L and circular cross-section with diameter ($d \ll L$) is held fixed between two walls as shown. The beam material has Young's modulus E and coefficient of thermal expansion α .



If the beam is slowly and uniformly heated, the temperature rise required to cause the beam to buckle is proportional to

- (A) d
- (B) d^2
- (C) d^3
- (D) d^4

[Ans. B]

As per Euler's theory

$$P_e = \frac{\pi^2}{l^2} EI \dots \dots \textcircled{1}$$

But $\frac{P}{A} = \sigma = (\alpha t)(E)$

$$P = \alpha t EA \dots \dots \textcircled{2}$$

Equating $\textcircled{1}$ and $\textcircled{2}$

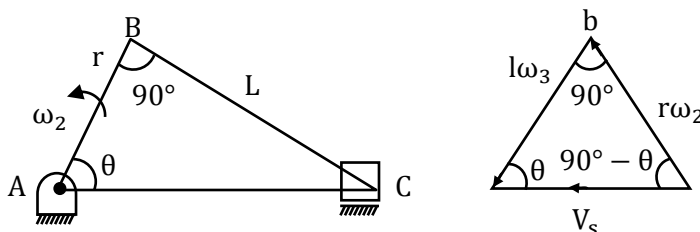
$$\frac{\pi^2}{l^2} EI = \alpha t EA$$

$$t \propto \frac{I}{A} \propto \frac{\frac{\pi}{64} d^4}{\frac{\pi}{4} d^2} \propto d^2$$

$$\therefore t \propto d^2$$

49. For an inline slider-crank mechanism, the lengths of the crank and connecting rod are 3 m and 4 m respectively. At the instant when the connecting rod is perpendicular to the crank, if the velocity of the slider is 1 m/s, the magnitude of angular velocity (upto 3 decimal points accuracy) of the crank is _____ radian/s.

[Ans. *]Range: 0.26 to 0.27



$$V_{\text{slider}} = 1 \text{ m/sec}, r = 3 \text{ m}, L = 4 \text{ m}$$

Refer the configuration diagram and velocity diagram

$$\tan \theta = \frac{L}{r} = \frac{4}{3} = 1.33 = n$$

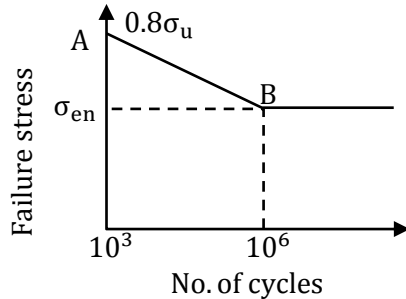
$$\theta = 53.13^\circ$$

$$V_{\text{slider}} = \frac{r\omega_2}{\sin \theta} = \frac{r\omega_2}{\sin 53.13} = 1.25r\omega_2$$

$$1 = 1.25 \times 3 \times \omega_2$$

$$\omega_2 = \frac{1}{3 \times 1.25} = 0.267 \text{ rad/sec}$$

50. A machine element has an ultimate strength (σ_u) of 600 N/mm², and endurance limit σ_{en} of 250 N/mm². The fatigue curve for the element on a log-log plot is shown below. If the element is to be designed for a finite life of 10000 cycles, the maximum amplitude of a completely reversed operating stress is _____ N/mm².



[Ans. *]Range: 370 to 390

$$s_{ut} = 600 \text{ MPa}, s_e = 250 \text{ MPa},$$

$$L = 10000 \text{ cycles}, s_f = ?$$

Basquin's equation,

$$A = s_f L^B$$

$$A = (0.8 \times 600) \times 10^{3B} \dots\dots \textcircled{1}$$

$$A = 250 \times 10^{6B} \dots\dots \textcircled{2}$$

Dividing equation $\textcircled{2}$ with equation $\textcircled{1}$

$$1 = 0.520 \times 10^{3B}$$

$$B = 0.094$$

$$\therefore A = 250 \times 10^{6 \times 0.094} = 916.09$$

$$\therefore 916.09 = s_f L^{0.094}$$

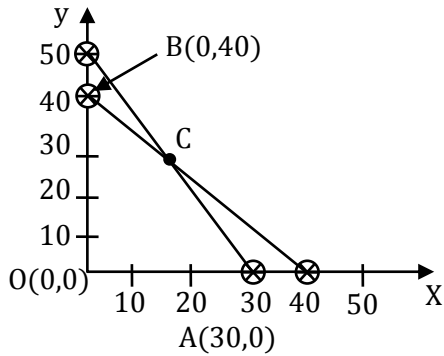
$$\therefore 916.09 = s_f \times 10^{4 \times 0.094}$$

$$s_f = 385.42 \text{ MPa}$$

51. Two models, P and Q, of a product earn profits of ₹ 100 and ₹ 80 per piece, respectively. Production time for P and Q are 5 hours and 3 hours, respectively, while the total production time available is 150 hours. For a total batch size of 40, to maximize profit, the number of units of P to be produced is _____.

[Ans. *]Range: 15 to 15

	P	Q	Availability
Production	5	3	150
Batch	1	1	40
Profit	100x	80y	



$$Z_{\max} = 100x + 80y$$

$$\text{s. t } 5x + 3y \leq 150$$

$$x + y = 40$$

$$x, y > 0$$

$$\frac{x}{30} + \frac{y}{50} = 1$$

$$\frac{x}{40} + \frac{y}{40} = 1$$

$$5x + 3y = 150$$

$$x + y = 40$$

$$x = 40 - y$$

$$5(40 - y) + 3y = 150$$

$$200 - 5y + 3y = 150$$

$$-2y = -50$$

$$y = 25$$

$$x + y = 40$$

$$x + 25 = 40$$

$$x = 15$$

Corner points are O(0, 0), A(30, 0), B(0, 40), C(15, 25)

$$Z_{\max} = 100x + 80y$$

$$Z_0 = 100 \times 0 + 80 \times 0 = 0$$

$$Z_A = 100 \times 30 + 80 \times 0 = 3000$$

$$Z_B = 100 \times 0 + 80 \times 40 = 3200$$

$$Z_C = 100 \times 15 + 80 \times 25 = 3500$$

Maximum at C

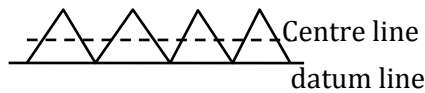
Number units of P = x = 15units

52. Assume that the surface roughness profile is triangular as shown schematically in the figure. If the peak to valley height is $20 \mu\text{m}$, the central line average surface roughness R_a (in μm) is



- (A) 5
(B) 6.67
(C) 10
(D) 20

[Ans. A]



$$R_t = 20$$

$$R_a = \frac{20 + 0}{2} = 10\mu\text{m}$$

53. A parametric curve defined by $x = \cos\left(\frac{\pi u}{2}\right)$, $y = \sin\left(\frac{\pi u}{2}\right)$ in the range $0 \leq u \leq 1$ is rotated about the X-axis by 360 degrees. Area of the surface generated is

(A) $\frac{\pi}{2}$

(B) π

(C) 2π

(D) 4π

[Ans. C]

$$x = \cos\left(\frac{\pi u}{2}\right) \text{ \& } y = \sin\left(\frac{\pi u}{2}\right), 0 \leq u \leq 1$$

$$\frac{dx}{du} = -\frac{\pi}{2} \sin\left(\frac{\pi u}{2}\right), \frac{dy}{du} = \frac{\pi}{2} \cos\left(\frac{\pi u}{2}\right)$$

$$\text{Surface area} = \int_{u=a}^b 2\pi y \frac{ds}{du} du$$

$$\text{where } \frac{ds}{du} = \sqrt{\left(\frac{dx}{du}\right)^2 + \left(\frac{dy}{du}\right)^2}$$

$$\frac{ds}{du} = \sqrt{\frac{\pi^2}{4} \sin^2\left(\frac{\pi u}{2}\right) + \frac{\pi^2}{4} \cos^2\left(\frac{\pi u}{2}\right)} = \frac{\pi}{2}$$

$$\begin{aligned} \therefore \text{Surface area} &= \int_0^1 2\pi y \left(\frac{\pi}{2}\right) du \\ &= \pi^2 \int_0^1 \sin\left(\frac{\pi u}{2}\right) du \\ &= \pi^2 \left\{ \frac{-\cos\left(\frac{\pi u}{2}\right)}{\frac{\pi}{2}} \right\}_0^1 \\ &= \pi^2 \times \frac{2}{\pi} \left\{ \left(-\cos\left(\frac{\pi}{2}\right)\right) - \left(-\cos(0)\right) \right\} \\ &= 2\pi \end{aligned}$$

54. A block of length 200 mm is machined by a slab milling cutter 34 mm in diameter. The depth of cut and table feed are set at 2 mm and 18 mm/minute, respectively. Considering the approach and the over travel of the cutter to be same, the minimum estimated machining time per pass is ___ minutes.

[Ans. *]Range: 12 to 12

$$l = 200 \text{ mm}, D = 34 \text{ mm}, d = 2 \text{ mm}$$

$$f_m = 18 \text{ mm/min,}$$

$$AP = OR = \text{Compulsory approach} = \sqrt{d(D-d)} = \sqrt{2(34-2)} = 8$$

$$L = \text{length of tool travel} = l + AP + OR$$

$$= 200 + 8 + 8 = 216 \text{ mm}$$

$$\text{Time/cut} = \frac{L}{f_m} = \frac{216}{18} = 12 \text{ min}$$

55. Two disks A and B with identical mass (m) and radius (R) are initially at rest. They roll down from the top of identical inclined planes without slipping. Disk A has all of its mass concentrated at the rim, while Disk B has its mass uniformly distributed. At the bottom of the plane, the ratio of velocity of the center of disk A to the velocity of the center of disk B is

(A) $\frac{\sqrt{3}}{4}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1

(D) $\sqrt{2}$

[Ans. A]

$$V = R\omega$$

$$I_A = mR^2, I_B = \frac{mR^2}{2}$$

$$\Rightarrow \frac{1}{2}mV_A^2 + \frac{1}{2}I_A\omega_A^2 = \frac{1}{2}mV_B^2 + \frac{1}{2}I_B\omega_B^2$$

$$\Rightarrow \frac{1}{2}m \times V_A^2 + \frac{1}{2}mR^2 \times \left(\frac{V_A}{R}\right)^2 = \frac{1}{2}mV_B^2 + \frac{1}{2} \times \frac{mR^2}{2} \times \left(\frac{V_B}{R}\right)^2$$

$$\Rightarrow \frac{1}{2}mV_A^2 + \frac{1}{2}mV_A^2 = \frac{1}{2}mV_B^2 + \frac{1}{4}mV_B^2$$

$$\Rightarrow mV_A^2 = \frac{3mV_B^2}{4}$$

$$\therefore \frac{V_A}{V_B} = \sqrt{\frac{3}{4}}$$