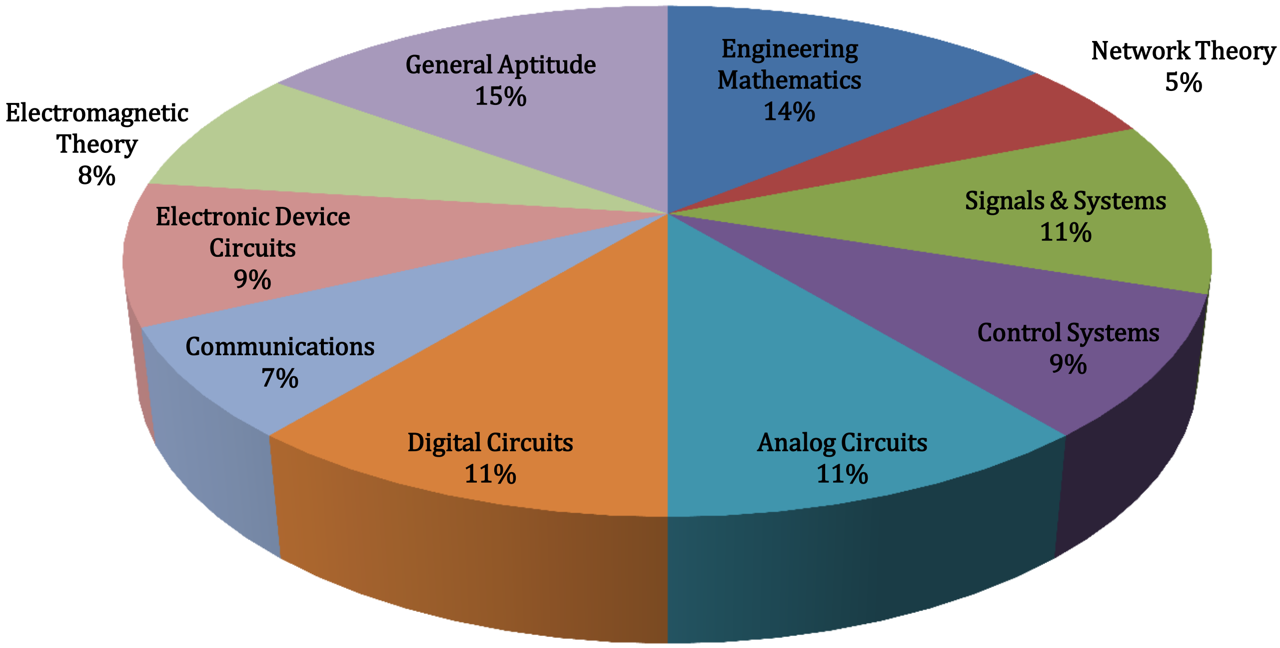
ANALYSIS OF GATE 2017
Electronics and Communication Engineering



ECE ANALYSIS-2017_4-Feb_Morning

SUBJECT	Ques. No.	Topics Asked in Paper(Memory Based)	Level of Toughness	Total Marks
Engineering Mathematics	1 Marks: 4 2 Marks: 5	Linear Algebra; Probability and Distribution; Complex Variables; Numerical Methods Differential Equations; Calculus	Tough	14
Network Theory	1 Marks: 1 2 Marks: 2	Sinusoidal Steady State Analysis; Transient/Steady State Analysis of RLC Circuits to DC Input	Easy	5
Signals & Systems	1 Marks: 3 2 Marks: 4	Linear Time Invariant(LTI) Systems; Introduction to Signals & Systems; Fourier Representation of Signals	Medium	11
Control Systems	1 Marks: 3 2 Marks: 3	Frequency Response Analysis Using Bode Plot and Nyquist Plot; Compensators & Controllers; State Variable Analysis; Stability & Routh Hurwitz Criterion; Stability & Routh Hurwitz Criterion	Easy	9
Analog Circuits	1 Marks: 3 2 Marks: 4	BJT & JFET Frequency Response; Operational Amplifiers & Its Applications; Feedback & Oscillator Circuits; Operational Amplifiers & Its Applications; Small Signal Modeling of BJT and FET; AC & DC Biasing-BJT and FET	Medium	11
Digital Circuits	1 Marks: 3 2 Marks: 4	Introduction to Microprocessor; Combinational and Sequential Digital Circuits; Boolean Algebra and Karnaugh Maps	Tough	11
Communications	1 Marks: 3 2 Marks: 2	Noise in Analog Modulation Digital Communications	Medium	7
Electronic Device Circuits	1 Marks: 3 2 Marks: 3	Semiconductor Theory; Transistor Theory(BJT,FET); P-N Junction Theory and Characteristics	Tough	9
Electromagnetic Theory	1 Marks: 2 2 Marks: 3	EM Wave Propagation; Antennas; Transmission Lines	Medium	8
General Aptitude	1 Marks: 5 2 Marks: 5	Numerical Ability; Verbal Ability	Easy	15
Total	65			100
Faculty Feedback	Majority of the question were direct concept based. Maths, Signal and Systems, analog Circuits, Digital Circuits and EDC weightage was comparatively high. Surprise was GA, it was comparatively tough as compared to the last year.			

GATE 2017 Examination

Electronics and Communication Engineering

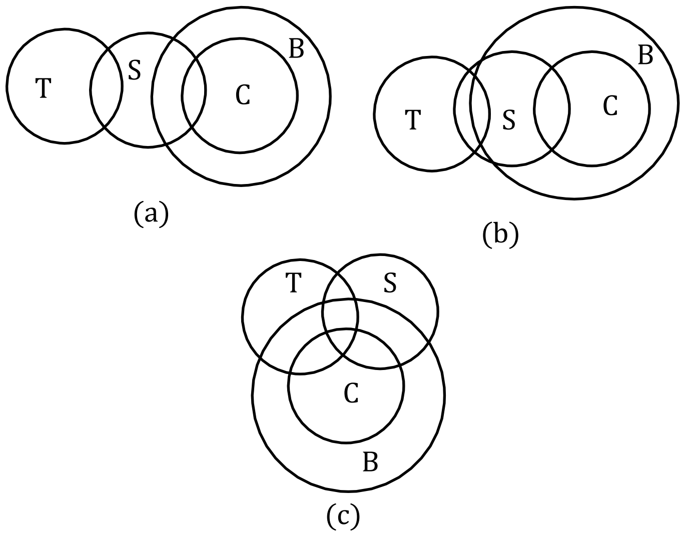
Test Date: 05/02/2017

Test Time: 9:00 AM to 12:00 PM

Subject Name: Electronics and Communication Engineering

Section: General Aptitude

1. Some tables are shelves. Some shelves are chairs. All chairs are benches. Which of the following conclusions can be deduced from the preceding sentences?
(i) At least one bench is a table
(ii) At least one shelf is a bench
(iii) At least one chair is a table
(iv) All benches are chairs
(A) Only i
(B) Only ii
(C) Only ii and iii
(D) Only iv
[Ans. B]
From given statements the following venn diagrams are possible
T = tables, S = shelves, C = chairs and B = benches



From all of the above diagrams, conclusion (ii) only deduced from the statements.

2. I ________ made arrangements had I _________ informed earlier.
(A) could have, been
(B) would have, being
(C) had, have
(D) had been, been
[Ans. A]
Conditional tense Type 3 – Past perfect (could have) + perfect conditional (had + V_3)

3. 40% of deaths on city roads may be attributed to drunken driving. The number of degrees needed to represent this as a slice of a pie chart is
(A) 120 (C) 160
(B) 144 (D) 212
[Ans. B]
Sum of angles in a pie chart = 360°
The relation between angle and percentage is
100 % = 360°
% = 3.6°
∴ 40% = ?
$40 \times 3.6 = 144°$

4. She has a sharp tongue and it can occasionally turn________
(A) hurtful (C) methodical
(B) left (D) vital
[Ans. A]
Hurtful: It is a supporting sentence. The word 'sharp tongue' strengthens the latter part of the sentence 'it can occasionally turn hurtful'

5. In the summer, water consumption is known to decrease overall by 25%. A water Board official states that in the summer household consumption decreases by 20% while other consumption increases by 70%.
Which of the following statements is correct?
(A) The ratio of household to other consumption is 8/17
(B) The ratio of household to other consumption is 1/17
(C) The ratio of household to other consumption is 17/8
(D) There are errors in the officials statement.
[Ans. D]
H = house hold consumption
P = other consumption
House hold consumption decreases by 20% $= \frac{80}{100}\mathrm{H}$
Other consumption increases by 70% $= \frac{170}{100}\mathrm{P}$
$$\frac{80\mathrm{H}}{100} + \frac{170\mathrm{P}}{100} = \frac{75}{100}(\mathrm{H} + \mathrm{P})$$
$80\mathrm{H} + 170\mathrm{P} = 75\mathrm{H} + 75\mathrm{P}$
$80\ \mathrm{H} - 75\ \mathrm{H} = 75\ \mathrm{P} - 170\mathrm{P}$
$5\ \mathrm{H} = -95\ \mathrm{P}$
There is a negative ratio so, there are errors in the official's statement.

6. "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, Or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country, I lived too near the

seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".
Here, the word 'antagonistic' is closest in meaning to

(A) impartial
(B) argumentative
(C) separated
(D) hostile

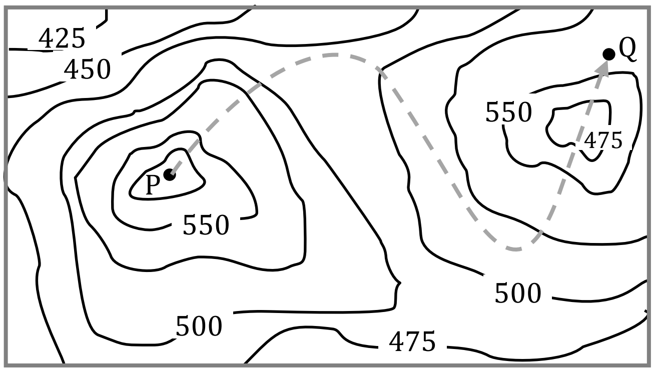
[Ans. D]
'Antagonistic' means showing dislike or opposition. So the word closest in meaning is 'hostile' (not friendly, having or showing unfriendly feelings, unpleasant or harsh)

7. Trucks (10 m long) and cars (5 m long) go on a single lane bridge. There must be a gap of at least 20 m after each truck and a gap of at least 15 m after each car. Trucks and cars travel at a speed of 36km/h. If cars and trucks go alternately, what is the maximum number of vehicles that can use the bridge in one hour?

(A) 1440
(B) 1200
(C) 720
(D) 600

[Ans. A]
Length of truck + gap required = 10 + 20 = 30 m
Length of car + gap required = 5 + 15 = 20 m
Total distance is need for truck and car for passing alternatively = 30 + 20 = 50 m
Given, speed $= 36 \text{ kmph} = 36 \times \dfrac{5}{18} = 10 \text{ m/sec}$
Let 'x' be the number of repetitions of (Truck + car) in one hour
$$\frac{50 \times x}{60 \times 60} = 10 \text{ m/sec}$$
$$x = \frac{10 \times 60 \times 60}{50} = 720 \text{ numbers of (Trucks + Cars)}$$
$\therefore$ The maximum number of vehicles $= 720 + 720 = 1440$

8. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region. Contour lines are shown at 25 m intervals in this plot. The path from P to Q is best described by



(A) Up-Down-Up-Down
(B) Down-Up-Down-Up
(C) Down-Up-Down
(D) Up-Down-Up

[Ans. C]
Contour lines can be observed to cross region with height from P to Q is as follows

(P)550 – 500 – 575 – 550(Q)

Down Up Down

∴ The path from P to Q is Down-Up-Down option (C) is satisfies this path

9. S, T, U, V, W, X, Y, and Z are seated around a circular table. T's neighbours are Y and V. Z is seated third to the left of T and second to the right of S. U's neighbours are S and Y; and T and W are not seated opposite each other. Who is third to the left of V?

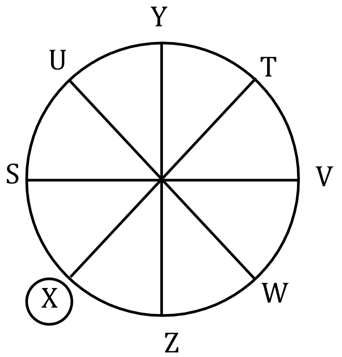
(A) X
(B) W
(C) U
(D) T

[Ans. A]

From the given data, eight persons are seated around a circular table as follows

Y T V | S U Y
(or) | (or)
V T Y | Y U S

S -- Z -- -- T



∴ X is third to the left of V

10. There are 3 Indians and 3 Chinese in a group of 6 people. How many subgroups of this group can we choose so that every subgroup has at least one Indian?

(A) 56
(B) 52
(C) 48
(D) 44

[Ans. A]

Sub group has at least one Indian means minimum one Indian and maximum three (or) more

Sub groups containing only Indians $= 3C_1 + 3C_2 + 3C_3 = 3 + 3 + 1 = 7$

In the sub group one Indian and remaining are Chinese

$= 3C_1[3C_1 + 3C_2 + 3C_3] = 3\,[3 + 3 + 1]$

$= 3 \times 7 = 21$

In the sub group two Indians and remaining are Chinese

$= 3C_2[3C_1 + 3C_2 + C_3] = 3\,[3 + 3 + 1]$

$= 3 \times 7 = 21$

In the sub group three Indians and remaining are Chinese

$= 3C_3[3C_1 + 3C_2 + 3C_3]$

$= 1[\,3 + 3 + 1\,]$

$= 7$

∴ Total number of sub groups $= 7 + 21 + 21 + 7 = 56$

Section: Technical

1. Let (X_1, X_2) be independent random variables, X_1 has mean 0 and variance 1, while X_2 has mean 1 and variance 4. The mutual information $I(X_1; X_2)$ between X_1 and X_2 in bits is________

[Ans. *] Range: 0.0 to 0.0

$$I(X_1, X_2) = H(X_1) - H\left(\frac{X_1}{X_2}\right)$$

$$= H(X_1) - H\,(X_1)\ [H\left(\frac{X_1}{X_2}\right) = H(X_1), \text{since } X_1\ \&\ X_2 \text{ are independent}]$$

$$= 0$$

2. Consider the following statements for continuous-time linear time invariant (LTI) systems.
   I. There is no bounded input bounded output (BIBO) stable system with a pole in the right half of the complex plane.
   II. There is no causal and BIBO stable system with a pole in the right half of the complex plane.

   Which one among the following is correct?

   (A) Both I and II are true
   (B) Both I and II are not true
   (C) Only I is true
   (D) Only II is true

   [Ans. D]

   I. For example consider a pole location at right half of complex plane, if it is anti-causal, ROC is left sided, and ROC includes jω axis, so It is a BIBO stable, so statement I is false.
   II. If a causal system having a pole on right side of s-plane it is compulsory unstable because ROC is not including jω-axis. So statement II is true.

3. Consider a stable system with transfer function

$$G(s) = \frac{s^p + b_1 s^{p-1} + \ldots + b_p}{s^q + a_1 s^{q-1} + \ldots + a_q}$$

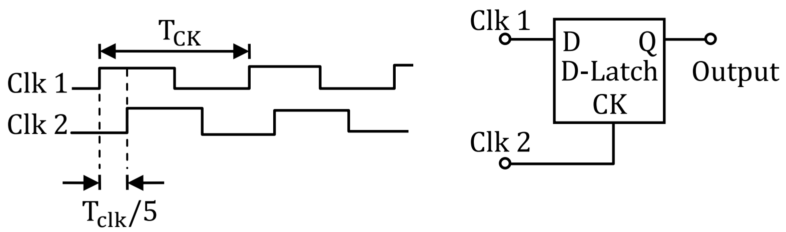
where $b_1 \ldots\ldots b_q$ and $a_1 \ldots\ldots a_q$ are real valued constants. The slope of the Bode log magnitude curve of G(s) converges to −60 dB/decade as $\omega \to \infty$. A possible pair of values for p and q is

(A) p = 0 and q = 3
(B) p = 1 and q = 7
(C) p = 2 and q = 3
(D) p = 3 and q = 5

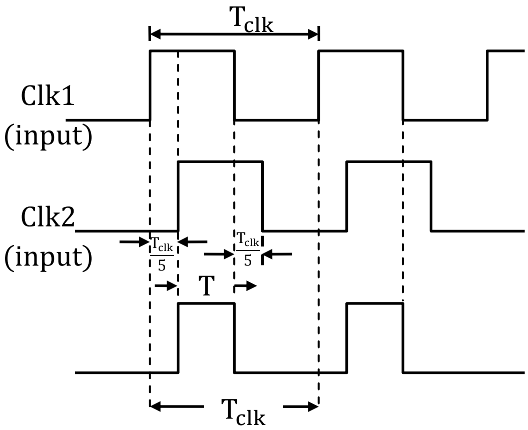
[Ans. A]

To get a slope of −60dB/decade, required $(p - q) = 3$

4. Consider the D-Latch shown in the figure, which is transparent when its clock input CK is high and has zero propagation delay. In the figure, the clock signal CLK1 has a 50% duty cycle and CLK2 is a one-fifth period delayed version of CLK1. The duty cycle at the output of the latch in percentage is________



[Ans. *] Range: 29.9 to 30.1



$$T = \frac{T_{clk}}{2} - \frac{T_{clk}}{5} = \frac{5T_{clk} - 2T_{clk}}{10} = \frac{3T_{clk}}{10}$$

So duty cycle is 30%

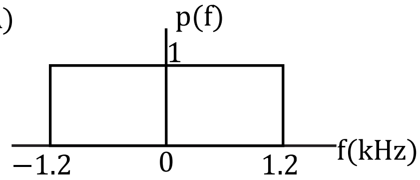
5. A bar of Gallium Arsenide (GaAs) is doped with Silicon such that the Silicon atoms occupy Gallium and Arsenic sites in the GaAs crystal. Which one of the following statements is true?
(A) Silicon atoms act as p-type dopants in Arsenic sites and n-type dopants in Gallium sites
(B) Silicon atoms act as n-type dopants in Arsenic sites and p-type dopants in Gallium sites
(C) Silicon atoms act as p-type dopants in Arsenic as well as Gallium sites
(D) Silicon atoms act as n-type dopants in Arsenic as well as Gallium sites
[Ans. A]
Substituting a Gallium site by a si atom produces a free electron so n-type
Substituting an Arsenic site by a si atom produces a hole so p-type.

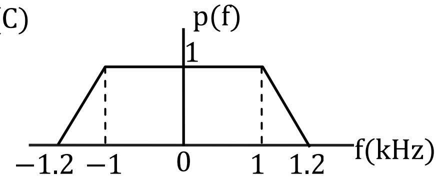
6. The rank of the matrix $M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$ is
(A) 0
(B) 1
(C) 2
(D) 3
[Ans. C]

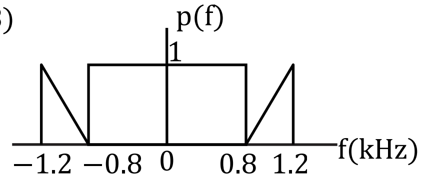
$$M = \begin{bmatrix} 5 & 10 & 10 \\ 1 & 0 & 2 \\ 3 & 6 & 6 \end{bmatrix}$$

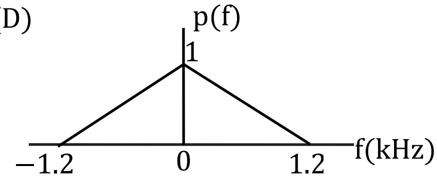
$\det(M) = 0$ ($\because$ first and third rows are proportional)
$\Rightarrow$ Rank of $M < 3$
Let $|M_1| = \begin{vmatrix} 5 & 10 \\ 1 & 0 \end{vmatrix}$ be a submatrix
$\det(M_1) = -10 \neq 0$
$\therefore$ Rank of $M = 2$

7. In a digital communication system, the overall pulse shape p(t) at the receiver before the sampler has the Fourier transform P(f). If the symbols are transmitted at the rate of 2000 symbols per second, for which of the following cases is the inter symbol interference zero?

(A)


(C)


(B)


(D)


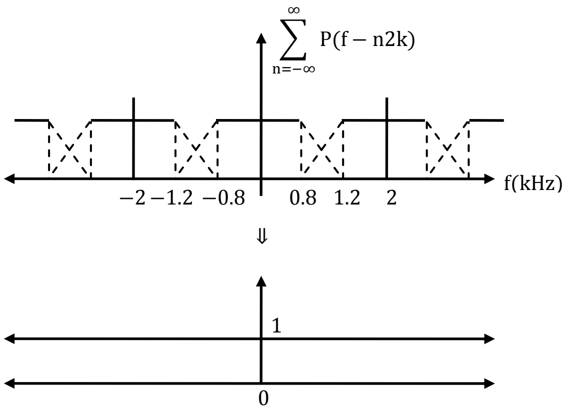
[Ans. B]

Given $f_s = \frac{1}{T_s} = 2\text{k symbols/sec}$

Condition for zero ISI is given by

$$\sum_{n=-\infty}^{\infty} P\left(f - \frac{n}{T_s}\right) = T_s(\text{constant})$$

The above condition is satisfied by only option (b)



$$\therefore \sum_{n=-\infty}^{\infty} P(f - n2k) = 1$$

Option (A)is correct if pulse duration is from -1 to $+1$

Option (C)is correct if the transition is from 0.8 to 1.2, -0.8 to -1.2

Option (D)is correct if the triangular duration is from -2 to $+2$

8. Which one of the following statements about differential pulse code modulation (DPCM) is true?

(A) the sum of message signal sample with its prediction is quantized

(B) The message signal sample is directly quantized, and its prediction is not used

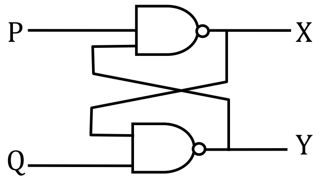
(C) The difference of message signal sample and a random signal is quantized

(D) The difference of message signal sample with its prediction is quantized

[Ans. D]

In DPCM the difference of the message signal sample value and the output of prediction filter block is quantized

9. In the latch circuit shown, the NAND gates have non-zero, but unequal propagation delays. The present input condition is: P = Q = '0', If the input condition is changed simultaneously to P = Q = '1', the outputs X and Y are

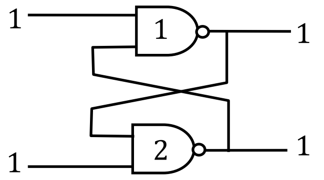


(A) X = '1', Y = '1'
(B) either X = '1', Y = '0' or X = '0', Y = '1'
(C) either X = '1', Y = '1' or X = '0', Y = '0'
(D) X = '0', Y = '0'

[Ans. B]

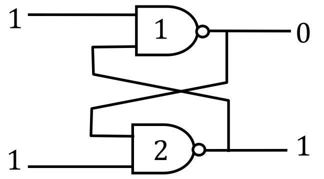
Take t_{pd} of gate 1 < t_{pd} of gate 2

If we take option (A):



Since output of 1st NAND is 1, it is input to second NAND, so output of second NAND has to be 0 (but given 1), so not satisfied.

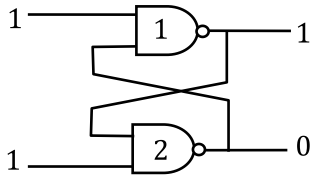
Take option (B):



Since output of 1st NAND is 0, it is input to second NAND, so output of second NAND has to be '1' (given also '1')

Hence satisfied

(or)



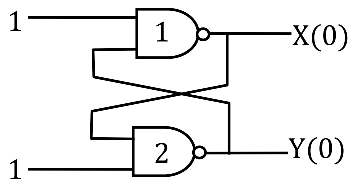
Output of 1st NAND is 1, it is input to second NAND, so output of second NAND has to be '0' (given also '0')

Hence satisfied

Take option C:

Already option (a) is not possible, hence 'c' is also not possible

Take option D:



Since output of first NAND is taken as '0', it is input to second, so output of second NAND has to be 1 (but given '0')
So not satisfied

10. Consider a single input single output discrete-time system with x[n] as input and y[n] as output, where the two are related as

$$y[n] = \begin{cases} n|x[n]|, & \text{for } 0 \le n \le 10 \\ x[n] - x[n-1], & \text{otherwise,} \end{cases}$$

Which one of the following statements is true about the system?

(A) It is causal and stable
(B) It is causal but not stable
(C) It is not causal but stable
(D) It is neither causal nor stable

[Ans. A]

The given system is

$$y(n) = \begin{cases} nx(n) & 0 \le n \le 10 \\ x(n) - x(n-1) & \text{otherwise} \end{cases}$$

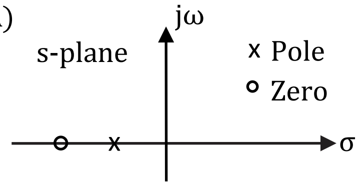
→Present output depends on present input and past input, so it is a causal system
→For a bounded input, bounded output yields, so it is a stable system

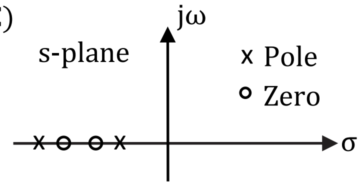
11. The Miller effect in the context of a Common Emitter amplifier explains
(A) an increase in the low-frequency cutoff frequency
(B) an increase in the high-frequency cutoff frequency
(C) a decrease in the low-frequency cutoff frequency
(D) a decrease in the high-frequency cutoff frequency

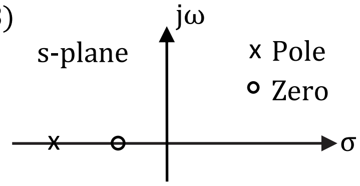
[Ans. D]

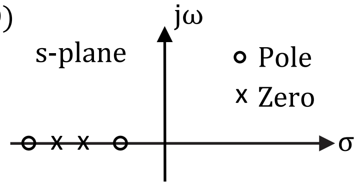
Due to miller effect the effective input Capacitance seen through the base terminal will increase which in turn reduces the higher cutoff frequency

12. Which of the following can be the pole-zero configuration of a phase-lag controller (lag compensator)?

(A)


(C)


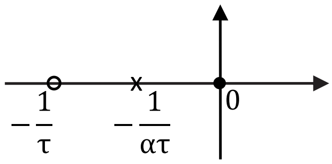
(B)


(D)


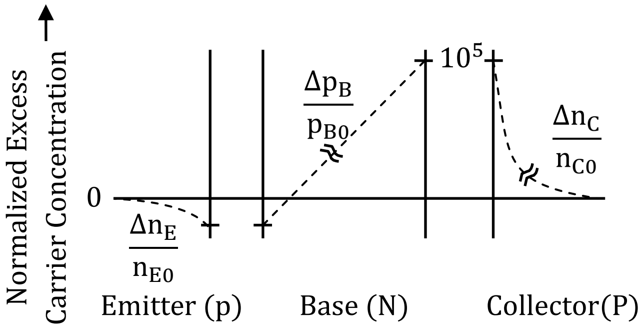
[Ans. A]

$$\text{TF for lag compensator} = \left(\frac{1 + \tau s}{1 + \alpha \tau s}\right)$$

For $\alpha > 1$ lag compensator



13. For a narrow base PNP BJT, the excess minority carrier concentrations (Δn_E for emitter, Δp_B base, Δn_C for collector) normalized to equilibrium minority carrier concentrations (n_{E0} for emitter, p_{B0} for base, n_{C0} for collector) in the quasi-neutral emitter, base and collector regions are shown below. Which one of the following biasing modes is the transistor operating in?



X and Y axes are not to scale

(A) Forward active
(B) Saturation
(C) Inverse active
(D) Cutoff

[Ans. C]

At C - B junction, due to FB a lot of holes are injected into the base from collector. At E-B junction, due to RB very few holes are injected into the base from Emitter (or)

At collector-Base junction, ratio of Excess minority carrier concentration to equilibrium minority carrier concentration is in order of 10^5 (very high) .This is possible when the junction is forward bias (injection)

At emitter-base junction, ratio of Excess minority carrier concentration to equilibrium minority carrier concentration is in order of 1 (negligible). This is possible when the junction is reverse Bias (no injection)

14. A periodic signal x(t) has a trigonometric Fourier Series expansion

$$x(t) = a_0 + \sum_{n=1}^{\infty}(a_n \cos n\omega_0 t + b_n \sin n\omega_0 t)$$

If $x(t) = -x(-t) = -x(t - \pi/\omega_0)$, we can conclude that

(A) a_n are zero for all n and b_n are zero for n even
(B) a_n are zero for all n and b_n are zero for n odd
(C) a_n are zero for n even and b_n are zero for n odd
(D) a_n are zero for n odd and b_n are zero for n even

[Ans. A]

Given that $x(t) = -x(-t)$ i.e odd signal , so the signal contains only b_n terms

And $x(t) = -x\left(-t - \frac{\pi}{\omega_0}\right)$

i.e Half wave symmetry , so the signal contains only odd harmonics.

So the resultant signal contains only b_n terms with odd harmonics.

15. The voltage of an electromagnetic wave propagating in a coaxial cable with uniform characteristic impedance is $V(l) = e^{-\gamma l + j\omega t}$ Volts, where l is the distance along the length of the cable in metres, $\gamma = (0.1 + j40)m^{-1}$ is the complex propagation constant, and $\omega = 2\pi \times 10^9$rad/s is the angular frequency. The absolute value of the attenuation in the cable in dB/metre is_______.

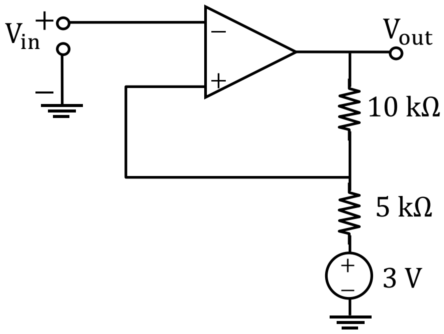
[Ans. *] Range: 0.85 to 0.88

Given $\gamma = (0.1 + j40)m^{-1} = \alpha + j\beta$

$\therefore \alpha = 0.1\,Np/m \quad (1\,N_p = 8.686\,dB)$

$\alpha_{dB} = 0.1 \times 8.686 = 0.8686 dB/m$

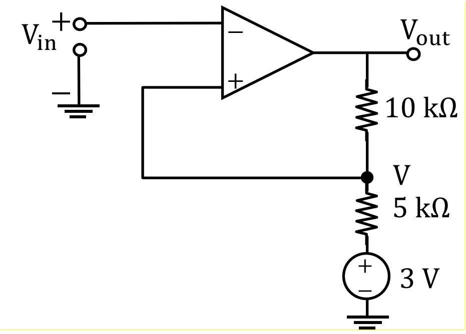
16. For the operational amplifier circuit shown, the output saturation voltages are $\pm$ 15V. The upper and lower threshold voltages for the circuit are, respectively.



(A) +5 V and − 5V
(B) +7 V and − 3V
(C) +3V and − 7V
(D) +3V and − 3V

[Ans. B]

Replying KCL at node (V)



$$\frac{V - V_{out}}{10K} + \frac{V - 3}{5k} = 0 \Rightarrow 3V - 6 = V_{out}$$

$$\Rightarrow V = \frac{V_{out} + 6}{3}$$

UTP: If $V_{out} = +V_{sat} = +15 \Rightarrow V_{UTP} = 7\,V$

LTP: If $V_{out} = -V_{sat} = -15 \Rightarrow V_{LTP} = -3V$

17. An $n^+ - n$ Silicon device is fabricated with uniform and non-degenerate donor doping concentrations of $N_{D1} = 1 \times 10^{18} cm^{-3}$ and $N_{D2} = 1 \times 10^{15} cm^{-3}$ corresponding to the n^+ and n regions respectively. At the operational temperature T, assume complete impurity ionization, $kT/q = 25$ mV, and intrinsic carrier concentration to be $n_i = 1 \times 10^{10} cm^{-3}$. What is the magnitude of the built-in potential of this device?

(A) 0.748 V
(B) 0.460 V
(C) 0.288 V
(D) 0.173 V

[Ans. D]

$$E_C - E_{F_{n+}} = KT \ln\left(\frac{N_C}{N_{D_1}}\right)$$

$$E_C - E_{F_n} = KT \ln\left(\frac{N_C}{N_{D_2}}\right)$$

$$E_{F_{n+}} - E_{F_n} = KT \ln\left(\frac{N_{D_1}}{N_{D_2}}\right)$$

$$V_o . q = KT \ln\left(\frac{N_{D1}}{N_{D2}}\right)$$

$$V_o = \frac{KT}{q} \ln\left(\frac{N_{D1}}{N_{D2}}\right)$$

$$= 25 \text{ mV} \ln(10^3)$$

$$= 0.173 \text{ V}$$

Another Method:

$$V_o = \frac{KT}{q} \ln\left[\frac{N_A N_D}{n_i^2}\right]$$

$$= \frac{KT}{q} \ln\left[\frac{N_{D_1}}{\left(\frac{n_i^2}{N_A}\right)}\right]$$

$$= \frac{KT}{q} \ln\left[\frac{N_{D_1}}{N_{D_2}}\right]$$

$$= 0.173 \text{ V}$$

18. A good trans-conductance amplifier should have

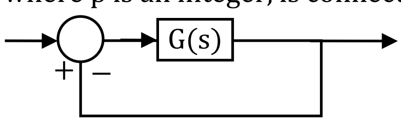
(A) high input resistance and low output resistance
(B) low input resistance and high output resistance
(C) high input and output resistances
(D) low input and output resistances

[Ans. C]

19. The open loop transfer function

$$G(s) = \frac{(s+1)}{s^p(s+2)(s+3)}$$

where p is an integer, is connected in unity feedback configuration as shown in the figure.



Given that the steady state error is zero for unit step input and is 6 for unit ramp input, the value of the parameter p is _______.

[Ans. *] Range: 0.99 to 1.01

To get steady state error zero for unit step input and 6 for unit ramp input, the type of the system is one.

20. Consider a wireless communication link between a transmitter and a receiver located in free space, with finite and strictly positive capacity. If the effective areas of the transmitter and the receiver antennas, and the distance between them are all doubled, and everything else remains unchanged, the maximum capacity of the wireless link

(A) increases by a factor of 2
(B) decreases by a factor of 2
(C) remains unchanged
(D) decreases by a factor of $\sqrt{2}$

[Ans. C]

Channel capacity $(C) = B\log\left(1+\frac{S}{N}\right)$

C depends on S

$$S = p_r(\text{received signal power}) = \frac{P_tG_tG_r}{\left(\frac{4\pi d}{\lambda}\right)^2} = \frac{P_t\left(\frac{4\pi}{\lambda^2}A_{et}\right)\left(\frac{4\pi}{\lambda^2}A_{er}\right)}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

Given that $(A_{et})_2 = 2(A_{et})_1$

$(A_{er})_2 = 2(A_{er})1$

$d_2 = 2d_1$

$P_r \propto \frac{A_{et}A_{er}}{d^2}$

$P_{r2} = P_{r1}$(received power remains unchanged)

$\therefore$ C remains unchanged

21. Three fair cubical dice are thrown simultaneously. The probability that all three dice have the same number of dots on the faces showing up is (up to third decimal place) ________.

[Ans. *] Range: 0.027 to 0.028

The probability that all three dice have the same number of dots on the faces showing up is

$$= \frac{6}{6^3} = \frac{1}{36} = 0.027$$

22. Consider the following statements about the linear dependence of the real valued functions $y_1 = 1, y_2 = x$ and $y_3 = x^2$. over the field of real numbers.

I. y_1, y_2 and y_3 are linearly independent on $-1 \le x \le 0$
II. y_1, y_2 and y_3 are linearly dependent on $0 \le x \le 1$
III. y_1, y_2 and y_3 are linearly independent on $0 \le x \le 1$
IV. y_1, y_2 and y_3 are linearly dependent on $-1 \le x \le 0$

Which one among the following is correct?

(A) Both I and II are true
(B) Both I and III are true
(C) Both II and IV are true
(D) Both III and IV are true

[Ans. B]

$y_1 = 1, y_2 = x, y_3 = x^2$

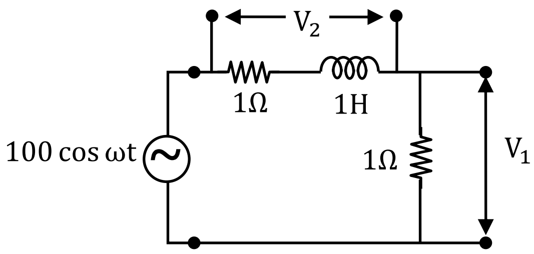
Let the Wronskian determinant be

$$W = \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & x \\ 0 & 1 \end{vmatrix} = 2 \neq 0$$

y_1, y_2 and y_3 are Linearly Independent for any $x \in (-\infty, +\infty)$

23. In the circuit shown, the positive angular frequency ω (in radians per second) at which the magnitude of the phase difference between the voltages V_1 and V_2 equals $\pi/4$ radians, is_______



[Ans. *] Range: 0.9 to 1.1

$$V_1(s) = \left(\frac{V_i(s)}{2+s}\right)1 \Rightarrow \frac{V_1(s)}{V_i(s)} = \frac{1}{s+2} \quad \rightarrow (1)$$

$$V_2(s) = \left(\frac{V_i(s)}{2+s}\right)(1+s)$$

$$\frac{V_2(s)}{V_i(s)} = \frac{1+s}{2+s} \rightarrow (2)$$

$$\frac{(1)}{(2)} \Rightarrow \frac{V_1(s)}{V_2(s)} = \frac{1}{s+1}$$

$$\Rightarrow \frac{V_1(j\omega)}{V_2(j\omega)} = \frac{1}{1+j\omega} = \frac{1}{\sqrt{1+\omega^2}} \angle -\tan^{-1}\omega$$

The given condition is $|-\tan^{-1}\omega| = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\omega = \frac{\pi}{4}$$

$$\Rightarrow \omega = \tan\frac{\pi}{4} = 1 \text{ rad/sec}$$

24. The clock frequency of an 8085 microprocessor is 5 MHz. If the time required to execute an instruction is 1.4 μs, then the number of T-states needed for executing the instruction is

(A) 1
(B) 6
(C) 7
(D) 8

[Ans. C]

$$f_{clk} = 5 \text{ MHz}; \; T_{clk} = \frac{1}{f_{clk}} = 0.2 \text{ μs}$$

$$(\text{Number of T-states}) \times (T_{clk}) = 1.4 \text{ μs}$$

$$\text{Number of T-states} = \frac{1.4 \text{ μs}}{0.2 \text{ μs}} = 7$$

25. Consider the 5×5 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

It is given that A has only one real eigen value. Then the real eigen value of A is

(A) -2.5
(B) 0
(C) 15
(D) 25

[Ans. C]

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 1 & 2 & 3 & 4 \\ 4 & 5 & 1 & 2 & 3 \\ 3 & 4 & 5 & 1 & 2 \\ 2 & 3 & 4 & 5 & 1 \end{bmatrix}$$

$|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 1-\lambda & 2 & 3 & 4 & 5 \\ 5 & 1-\lambda & 2 & 3 & 4 \\ 4 & 5 & 1-\lambda & 2 & 3 \\ 3 & 4 & 5 & 1-\lambda & 2 \\ 2 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$C_1 \rightarrow C_1 + C_2 + C_3 + C_4 + C_5$

$$\Rightarrow \begin{vmatrix} 15-\lambda & 2 & 3 & 4 & 5 \\ 15-\lambda & 1-\lambda & 2 & 3 & 4 \\ 15-\lambda & 5 & 1-\lambda & 2 & 3 \\ 15-\lambda & 4 & 5 & 1-\lambda & 2 \\ 15-\lambda & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

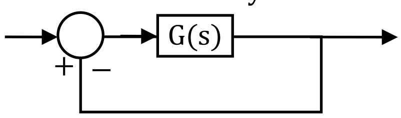
$$\Rightarrow (15-\lambda) = \begin{vmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1-\lambda & 2 & 3 & 4 \\ 1 & 5 & 1-\lambda & 2 & 3 \\ 1 & 4 & 5 & 1-\lambda & 2 \\ 1 & 3 & 4 & 5 & 1-\lambda \end{vmatrix} = 0$$

$\Rightarrow 15 - \lambda = 0 \therefore \lambda = 15$

26. A linear time invariant (LTI) system with the transfer function

$$G(s) = \frac{K(s^2 + 2s + 2)}{(s^2 - 3s + 2)}$$

is connected in unity feedback configuration as shown in the figure .

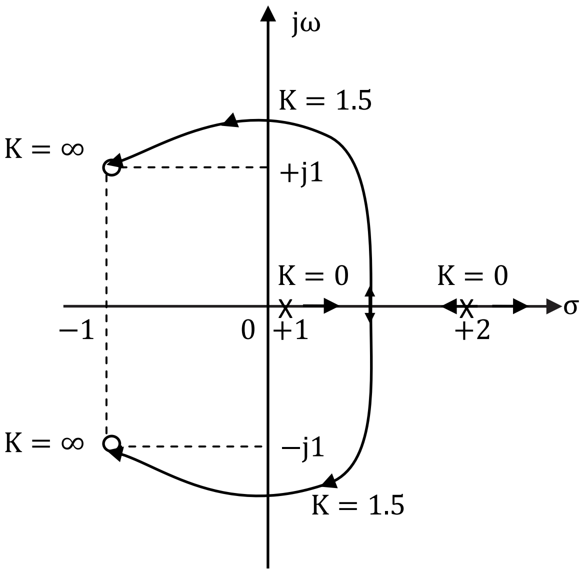


For the closed loop system shown, the root locus for $0 < K < \infty$ intersects the imaginary axis for $K = 1.5$. The closed loop system is stable for

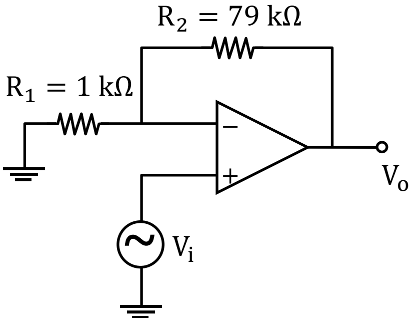
(A) $K > 1.5$
(B) $1 < K < 1.5$
(C) $0 < K < 1$
(D) no positive value of K

[Ans. A]

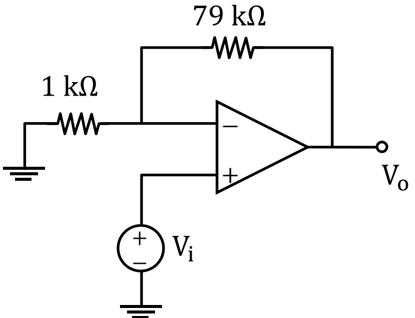
For CL system stability system gain is greater than 1.5



27. The amplifier circuit shown in the figure is implemented using a compensated operational amplifier (op-amp), and has an open-loop voltage gain, $A_0 = 10^5$V/V and an open-loop cutoff frequency, $f_c = 8$ Hz . The voltage gain of the amplifier at 15 kHz in V/V, is________



[Ans. *] Range: 43.3 to 45.3



Method 1:

$$A(s) = \frac{A_o}{1 + \frac{s}{\omega_p}} = \frac{10^5}{1 + \frac{s}{2\pi f_c}}$$

$$V_- = V_o \times \frac{1}{80} = \frac{V_o}{80}$$

$$V_o = \left(V_I - \frac{V_o}{80}\right) A(s)$$

$$V_o\left(I+\frac{A(s)}{80}\right)=V_I A(s)$$

$$\frac{V_o}{V_I}=\frac{A(s)}{1+\frac{A(s)}{80}}=\frac{\frac{10^5}{1}+\frac{s}{\omega_c}}{1+\frac{10^5}{80}\cdot\frac{1}{1+\frac{s}{\omega_c}}}$$

$$=\frac{80\times10^5}{80+\frac{80s}{\omega_c}+10^5}$$

$$\frac{V_o}{V_1}=\frac{80\times10^5}{10^5+80+\frac{80s}{\omega_c}}$$

$$|A|=\frac{80\times10^5}{\sqrt{(80+10^5)^2+\left(\frac{80\times15\times10^3}{8}\right)^2}}\approx 44.36$$

Method II:

$$A_{CL}=\frac{A/1+A\beta}{1+jf/f_c(1+A\beta)}$$

$$f_c(1+A\beta)=8\times\left(1+10^5\times\frac{1}{80}\right)\approx 10\text{ kHz}$$

$$A_{CL}\approx\frac{80}{1+jf/10K}$$

$$|A_{CL}|_{(f=15k)}=\left|\frac{80}{1+j\frac{15k}{10k}}\right|=\frac{80}{\sqrt{1+(1.5)^2}}=44.37$$

28. Starting with $x=1$, the solution of the equation $x^3+x=1$, after two iterations of Newton-Raphson's method (up to two decimal places) is ________

[Ans. *] Range: 0.65 to 0.72

Let $f(x)=x^3+x-1, x_0=1$

$f'(x)=3x^2+1$

The first approximation is

$$x_1=x_0-\frac{f(x_0)}{f'(x_0)}=1-\frac{f(1)}{f'(1)}$$

$$=1-\frac{1}{4}=\frac{3}{4}=0.75$$

The second approximation is

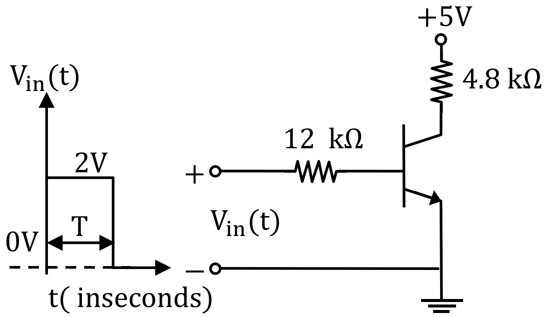
$$x_2=x_1-\frac{f(x_1)}{f'(x_1)}$$

$$=0.75-\frac{f(0.75)}{f'(0.75)}$$

$$=0.75-\frac{0.1718}{2.6875}$$

$x_2=0.6861$

29. In the figure shown, the npn transistor acts as a switch.



For the input $V_{in}(t)$as shown in the figure, the transistor switches between the cut-off and saturation regions of operation, when T is large. Assume collector-to-emitter voltage at saturation $V_{CE(sat)} = 0.2V$ and base-to-emitter voltage $V_{BE} = 0.7V$.The minimum value of the common-base current gain (α) of the transistor for the switching should be________

[Ans. *] Range: 0.89 to 0.91

$$I_B = \frac{2-0.7}{12k\Omega} = 0.108 \text{ mA}$$

$$I_{C-sat} = \frac{5-0.2}{4.8 \text{ k}\Omega} = 1 \text{ mA}$$

$$I_k = I_{C-sat} + I_B$$

$$= 1 \text{ mA} + 0.108 \text{ mA}$$

$$= 1.108 \text{ mA}$$

For saturation

$$I_{C-sat} < \alpha I_E$$

$$\alpha > \frac{I_{C-sat}}{IE}$$

$$\alpha > \frac{1 \text{ mA}}{1.108 \text{ mA}}$$

$$\alpha > 0.902$$

30. The Nyquist plot of the transfer function

$$G(s) = \frac{K}{(s^2+2s+2)(s+2)}$$

does not encircle the point (−1+j0) for K = 10 but does encircle the point (−1 + j0) for K = 100. Then the closed loop system (having unity gain feedback) is

(A) stable for K = 10 and stable for K = 100
(B) stable for K = 10 and unstable for K = 100
(C) unstable for K = 10 and stable for K = 100
(D) unstable for K = 10 and unstable for K = 100

[Ans. B]

For given system, $P = 0$

For stability $N = 0$

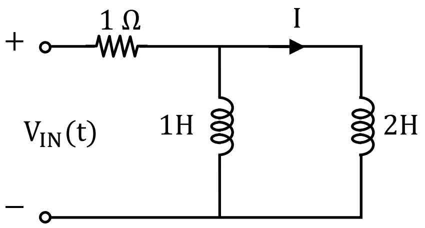
For $k = 10$, No encirclements about $(-1, j0)$. Hence the system is stable

For $K = 100$, encircles the point $(-1, j0)$. Hence the system is unstable.

31. In the circuit shown the voltage $V_{IN}(t)$ is described by:

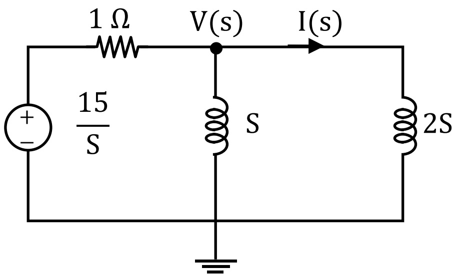
$$V_{IN}(t) = \begin{cases} 0 & \text{for } t < 0 \\ 15 \text{ Volts}, & \text{for } t \geq 0 \end{cases}$$

where 't' is in seconds. The time (in seconds) at which the current I in the circuit will reach the value 2 Ampere is _______



[Ans. *] Range: 0.30 to 0.40

By applying the Laplace transform with initial conditions



Nodal in S-D⇒

$$\frac{V(s) - \frac{15}{s}}{1} + \frac{V(s)}{s} + \frac{V(s)}{2s} = 0$$

$$\Rightarrow V(s) = \frac{30}{2s+3}$$

$$\rightarrow I(s) = \frac{V(s)}{z(s)} = \frac{V(s)}{2(s)} = \frac{30}{2s(2s+3)}$$

$$\Rightarrow I(s) = 5\left[\frac{1}{5} \quad \frac{1}{s+\frac{3}{2}}\right]$$

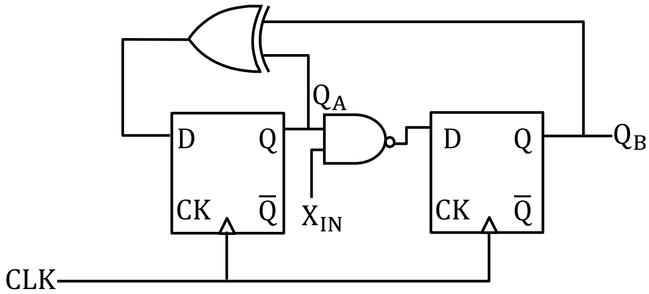
$$\Rightarrow i(t) = 5\left(1 - e^{-\frac{3t}{2}}\right) \text{ A for } 0 \leq t \leq \infty$$

When $i(t) = 2$A then

$$2 = 5\left(1 - e^{-\frac{3t}{2}}\right)$$

$$\Rightarrow t = \frac{2}{3}\log_e \frac{5}{3} = 0.3405 \text{ sec}$$

32. A finite state machine (FSM) is implemented using the D flip-flops A and B and logic gates, as shown in the figure below. The four possible states of the FSM are $Q_AQ_B = 00, 01, 10,$ and 11.



Assume that X_{IN} is held at a logic level throughout the operation of the FSM. When the FSM is initialized to the state $Q_AQ_B = 00$ and clocked, after a few clock cycles , it starts cycling through

(A) all of the four possible states if $X_{IN} = 1$
(B) three of the four possible states if $X_{IN} = 0$
(C) only two of the four possible states if $X_{IN} = 1$
(D) only two of the possible states if $X_{IN} = 0$

[Ans. D]

If $X_{IN} = 0$

$D_B = \overline{Q_A \cdot 0} = 1$

Clk	$D_A = Q_A \oplus Q_B$	$D_B = 1$	Q_A	Q_B
0	-	-	0	0
1	0	1	0	1
2	1	1	1	1
3	0	1	0	1

Thus number of possible states are two

If $X_{IN} = 1 \Rightarrow D_B = \overline{Q_A \cdot 1} = \overline{Q_A}$

Clk	$D_A = Q_A \oplus Q_B$	$D_B = \overline{Q}_A$	Q_A	Q_B
0			0	0
1	0	1	0	1
2	1	1	1	1
3	0	1	0	1

Thus number of possible states are three

33. Let $I = \int_c (2z\,dx + 2y\,dy + 2x\,dz)$ where x, y , z are real , and let C be the straight line segment from point A: (0,2, 1) to point B: (4,1,–1) . The value of I is _______

[Ans. *] Range: -11.1 to -10.9

Given

$$I = \int_C (2z\,dx + 2y\,dy + 2x\,dz)$$

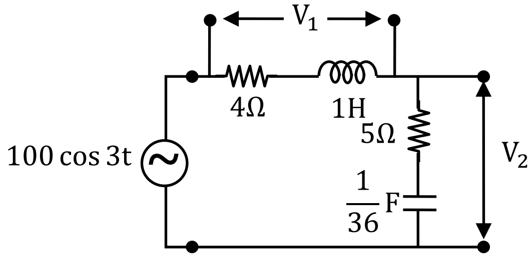
$$I = \int_A^B [2d(xz) + 2ydy]$$

$$= \int_{(0,2,1)}^{(4,1,-1)} [2d(xz) + 2ydy]$$

$$= [2xz + y^2]_{(0,2,1)}^{(4,1,-1)}$$
$$= (-8+1) - (4)$$
$$= -11$$

34. The figure shows an RLC circuit excited by the sinusoidal voltage 100cos(3t) Volts, where t is in seconds.

The ratio $\dfrac{\text{amplitude of } V_2}{\text{amplitude of } V_1}$ is ________



[Ans. *] Range: 2.55 to 2.65

$$\rightarrow V_1(s) = \left(\frac{V_i(s)}{4+s+5+\frac{36}{s}}\right)(4+s)$$

$$\Rightarrow \frac{V_1(s)}{V_i(s)} = \left(\frac{s+4}{s+\frac{36}{s}+9}\right) \ldots\ldots\ldots (1)$$

$$\rightarrow V_2(s) = \left[\frac{V_i(s)}{4+s+5+\frac{36}{s}}\right]\left(5+\frac{36}{s}\right)$$

$$\Rightarrow \frac{V_2(s)}{V_i(s)} = \left[\frac{5+\frac{36}{s}}{s+\frac{36}{s}+9}\right] \ldots\ldots\ldots (2)$$

$$\frac{2}{1} = \frac{V_2(s)}{V_1(s)} = \left(\frac{5+\frac{36}{s}}{s+4}\right)$$

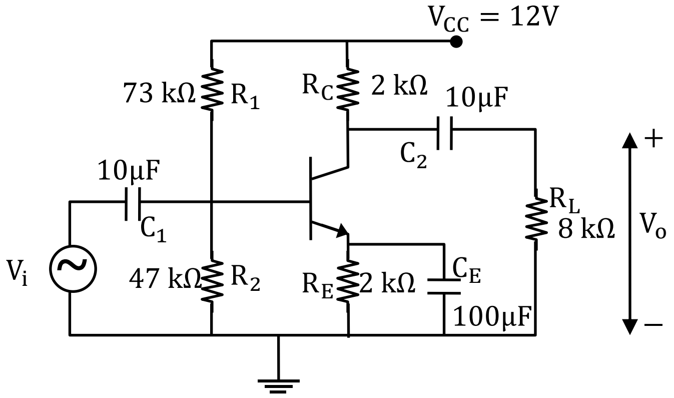
$$\Rightarrow \left|\frac{V_2(j\omega)}{V_1(j\omega)}\right| = \left|\frac{5+\frac{36}{j\omega}}{4+j\omega}\right|$$

$$\rightarrow \left|\frac{V_2(j\omega)}{V_1(j\omega)}\right|_{\omega=3\text{rad/sec}} = \frac{\left|5+\frac{36}{j3}\right|}{|4+j3|}$$

$$= \frac{|5-j12|}{|4+j3|}$$

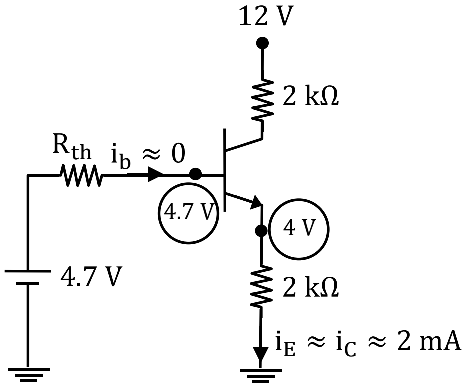
$$= \frac{13}{5} = 2.6$$

35. For the DC analysis of the common-Emitter amplifier shown, neglect the base current and assume that the emitter and collector currents are equal . Given that $V_T = 25Mv$, $V_{BE} = 0.7V$, and the BJT output resistance r_0 is practically infinite. Under these conditions the midband voltage gain magnitude, $A_v = |v_o/v_i|V/V$, is ______



[Ans. *] Range: 127.0 to 129.0

DC Analysis:



AC Analysis :

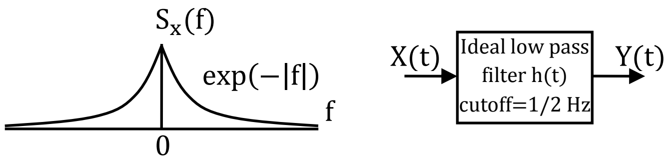
$$|A_V| = |g_m(R_C||R_L)|$$

$$= \frac{2}{25} \times (2k||8k) = 128$$

36. Let X(t) be a wide sense stationary random process with the power spectral density $S_x(f)$ as shown in figure (a), where f is in Hertz (Hz) . The random process X(t) is input to an ideal low pass filter with the frequency response

$$H(t) = \begin{cases} 1, & |f| \le \frac{1}{2}\text{Hz} \\ 0, & |f| > \frac{1}{2}\text{Hz} \end{cases}$$

As shown in Figure (b). The output of the low pass filter is Y(t).



(a) (b)

Let E be the expectation operator and consider the following statements:

I. $E(X(t)) = E(Y(t))$
II. $E\left(X^2(t)\right) = E(Y^2(t))$
III. $E\left(Y^2(t)\right) = 2$

Select the correct option:

(A) only I is true
(B) only II and III are true
(C) only I and II are true
(D) only I and III are true

[Ans. A]

I. **E[x(t)] = E[y(t)]**

Proof:

$E[y(t)] = H(0)E[x(t)]$

$= E[x(t)][\because H(0) = 1]$

II. $[\mathbf{x^2(t)}] \neq \mathbf{E}[\mathbf{y^2(t)}]$

Proof

$$E[y^2 9t)] = \int_{-\infty}^{\infty} s_y(f)df = \int_{-\infty}^{\infty} s_x(f)|H(f)|^2\,df = \int_{-\frac{1}{2}}^{\frac{1}{2}} s_x(f)df = 2 - 2e^{-0.5} \ldots\ldots\ldots (1)$$

$$E[x^2(t)] = \int_{-\infty}^{\infty} s_x(f)df = 2 \ldots\ldots..(2)$$

$(1) \neq (2)$

III. $\mathbf{E}\left[\mathbf{y^2(t)}\right] \neq \mathbf{2}$

Proof:

From (II)

$E\,[x^2(t)] = 2$ but $E[y^2(t)] \neq 2$

37. A continuous time signal $x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$, where t is in seconds, is the input to a linear time invariant (LTI) filter with the impulse response

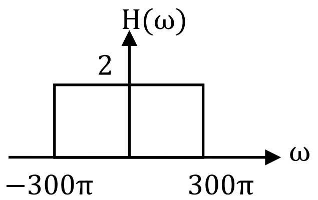
$$h(t) = \begin{cases} \dfrac{2\sin(300\pi t)}{\pi t} & t \neq 0 \\ 600, & t = 0 \end{cases}$$

Let $y(t)$ be the output of this filter. The maximum value of $|y(t)|$ is ___________

[Ans. *] Range: 7.90 to 8.10

$x(t) = 4\cos(200\pi t) + 8\cos(400\pi t)$

$$h(t) = \begin{cases} \dfrac{2\sin(300\pi t)}{\pi t}; & t \neq 0 \\ 600; & t = 0 \end{cases}$$



The input signal frequencies are 100, 200 Hz

The output signal is $= 2 \times 4\cos(200\pi t) = 8\cos(200\pi t)$

The maximum value $|y(t)|$ is 8

38. Let $f(x) = e^{x+x^2}$ for real x. From among the following. Choose the Taylor series approximation of f (x) around x = 0, which includes all powers of x less than or equal to 3.

(A) $1 + x + x^2 + x^3$

(B) $1 + x + \frac{3}{2}x^2 + x^3$

(C) $1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$

(D) $1 + x + 3x^2 + 7x^3$

[Ans. C]

Given $f(x) = e^{x+x^2}$ ……..①

We know that $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots..$

$$e^{x+x^2} = 1 + (x + x^2) + \frac{(x + x^2)^2}{2!} + \frac{(x + x^2)^3}{3!}$$

$$= 1 + x + x^2 + \frac{(x^2 + 2x^3)}{2} + \frac{x^3}{6}$$

$$= 1 + x + \frac{3}{2}x^2 + \frac{7}{6}x^3$$

39. An optical fiber is kept along the $\hat{z}$ direction. The refractive indices for the electric fields along $\hat{x}$ and $\hat{y}$ direction in the fiber are $n_x = 1.5000$ and $n_y = 1.5001$, respectively ($n_x \neq n_y$ due to the imperfection in the fiber cross-section). The free space wavelength of a light wave propagating in the fiber is 1.5 μm. If the light wave is circularly polarized at the input of the fiber, the minimum propagation distance after which it becomes linearly polarized, in centimeters, is _______.

[Ans. *] Range: 0.36 to 0.38

For to have linear polarization, phase difference has to be 0° or 180°. Given the light wave is circularly polarized that is initial phase difference is 90°.

So

$$\beta_1 z \sim \beta_2 z = \frac{\pi}{2}$$

$$\Rightarrow \frac{\omega}{v_{px}} z \sim \frac{\omega}{v_{py}} z = \frac{\pi}{2}$$

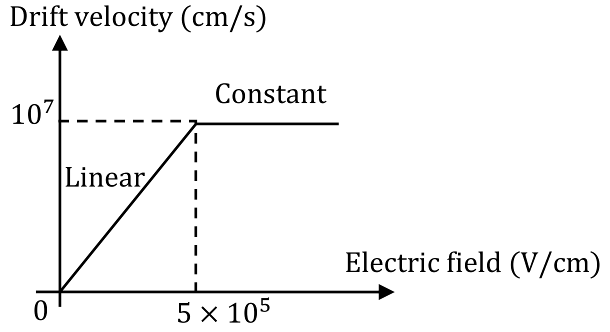
$$\Rightarrow 2\pi f\left(\frac{n_x}{c} \sim \frac{n_y}{c}\right) z = \frac{\pi}{2}\left(v_{px} = \frac{c}{n_x} \text{and } v_{py} = \frac{c}{n_y}\right)$$

$$\Rightarrow \frac{2\pi f}{c}(n_x \sim n_y)\, z = \frac{\pi}{2}$$

$$\Rightarrow \frac{2\pi}{\lambda}(n_x \sim n_y)\, z = \frac{\pi}{2}$$

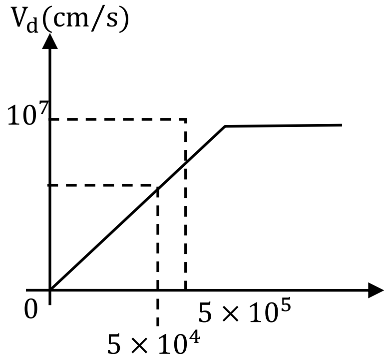
$$\Rightarrow z = \frac{\pi}{2} \times \frac{\lambda}{2\pi}(n_x \sim n_y) = \frac{\lambda}{4}(n_x \sim n_y) = \frac{1.5 \times 10^{-6}}{4 \times 0.0001} = 0.375 \text{ cm}$$

40. The dependence of drift velocity of electrons on electric field in a semiconductor is shown below. The semiconductor has a uniform electron concentration of $n = 1 \times 10^{16}\ \text{cm}^{-3}$ and electronic charge $q = 1.6 \times 10^{-19}$. If a bias of 5V is applied across a 1μm region of this semiconductor, the resulting current density in this region, in kA/cm^2, is _______.



[Ans. *] Range: 1.5 to 1.7

$$E = \frac{V}{d} = \frac{5}{10^{-6}\text{m}} = 5 \times 10^4 \text{V/cm}$$



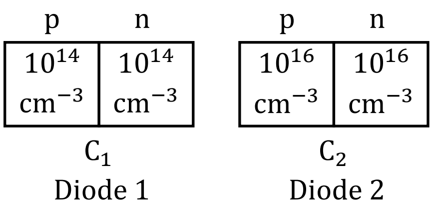
At $E = 5 \times 10^4 \text{V/cm} \Rightarrow V_d = 10^6 \text{cm/s}$

$J = \sigma E = n\, q\mu_n E = n\, q\, V_d$

$= 10^{16} \times 1.6 \times 10^{-19} \times 10^6$

$= 1.6 \text{ KA/cm}^2$

41. As shown, two Silicon (Si) abrupt p-n junction diodes are fabricated with uniform donor doping concentrations of $N_{D1} = 10^{14}\text{cm}^{-3}$ and $N_{D2} = 10^{16}\text{cm}^{3-}$ in the n-regions of the diodes, and uniform acceptor doping concentrations of $N_{A1} = 10^{14}\text{cm}^{-3}$ and $N_{A2} = 10^{16}\text{ cm}^{-3}$ in the p-regions of the diodes, respectively. Assuming that the reverse bias voltage is $\gg$ built-in potentials of the diodes, the ratio C_2/C_1 of their reverse bias capacitances for the same applied reverse bias, is



[Ans. *] Range: 10.0 to 10.0

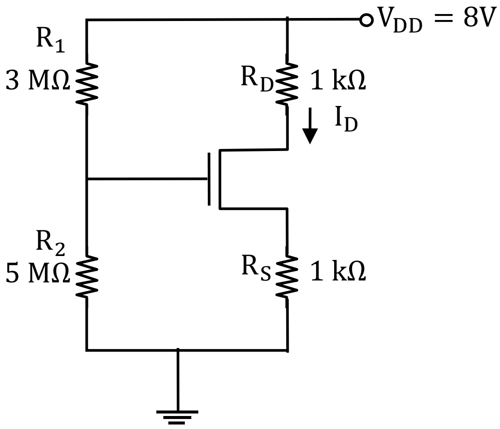
$$C = \frac{\varepsilon A}{\omega}$$

For abrupt junction, $\omega \sqrt{\frac{2\varepsilon(V_0 + V_R)}{q}\left(\frac{1}{N_A} + \frac{1}{N_D}\right)}$

Since $V_o \ll V_R \Rightarrow V_o + V_R = V_R$

$$\frac{C_2}{C_1} = \sqrt{\frac{\frac{N_{A_2}N_{D_2}}{V_{R_2}(N_{A_2}+N_{D_2})}}{\frac{N_{A_1}N_{D_1}}{V_{R_1}(N_{A_1}+N_{D_1})}}} = \sqrt{\frac{\frac{10^{32}}{2\times10^{16}}}{\frac{10^{28}}{2\times10^{14}}}} = \sqrt{\frac{10^{16}}{10^{14}}} = \sqrt{100} = 10$$

42. For the circuit shown, assume that the NMOS transistor is in saturation. Its threshold voltage $V_{tn} = 1V$ and its trans-conductance parameter $\mu_n C_{ox}\left(\frac{W}{L}\right) = 1mA/V^2$. Neglect channel length modulation and body bias effects. Under these conditions the drain current I_D in mA is_______



[Ans. *] Range: 1.9 to 2.1

$$V_G = 8 \times \frac{5}{5+3} = 5V$$
$$V_s = I_D \cdot 1\,k\Omega$$
$$V_{GS} = 5 - I_D 1k\Omega$$
$$I_D = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{GS} - V_{tn})^2$$
$$I_D = \frac{1}{2} \times 10^{-3}(5 - I_D \cdot 1k - 1)^2$$
$$2000I_D = (4 - I_D \cdot 1k)^2$$
$$2000I_D = 16 + I_D^2 \cdot 10^6 - 8I_D \cdot 1k$$
$$10^6 I_D^2 - 10^4 I_D + 16 = 0$$
$$I_D = \frac{10^4 \pm \sqrt{10^8 - (4\times16\times10^6)}}{2\times10^6}$$
$$\frac{10^4 \pm 6\times10^3}{2\times10^6} = 8\,mA/2\,mA$$
$I_D = 8$ mA when $V_s = 8V$ (Not possible)
$I_D = 2$ mA when $V_s = 2$ V (possible)

43. Which one of the following gives the simplified sum of products expression for the Boolean function $F = m_0 + m_2 + m_3 + m_5$, where m_0, m_2, m_3 and m_5 are minterms corresponding to the inputs A, B, and C with A as the MSB and C as the LSB?

(A) $\bar{A}B + \bar{A}\bar{B}\bar{C} + A\bar{B}C$
(B) $\bar{A}\bar{C} + \bar{A}B + A\bar{B}C$
(C) $\bar{A}\bar{C} + A\bar{B} + A\bar{B}C$
(D) $\bar{A}BC + \bar{A}\bar{C} + A\bar{B}C$

[Ans. B]

$F = \sum m(0,2,3,5)$

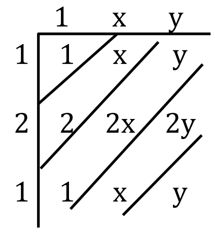
	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$
$\bar{A}$	1		1	1
A		1		

$F = A\bar{B}C + \bar{A}\bar{C} + \bar{A}B$

$= \bar{A}\bar{C} + \bar{A}B + A\bar{B}C$

44. Two discrete-time signals x [n] and h [n] are both non-zero only for n = 0,1,2,and are zero otherwise . It is given that x[0] = 1, x[1] = 2, x[2] =1, h[0] = 1.
Let y[n] be the linear convolution of x [n] and h [n] . Given that y[1] = 3 and y[2] = 4, the value of the expression (10y[3] + y[4]) is_______

[Ans. *] Range: 31.00 to 31.00



$x(n) = \{1,2,1\}$

$h(n) = \{1, x, y\}$

$y(n) = x(n) * h(n)$

$y(n) = \{1, 2 + x, 2x + y + 1, x + 2y, y\}$

$y(1) = 3 = 2 + x \Rightarrow x = 1$

$y(2) = 4 = 2x + y + 1 \Rightarrow y = 1$

$y(n) = \{1, 3, 4, 3, 1\}$

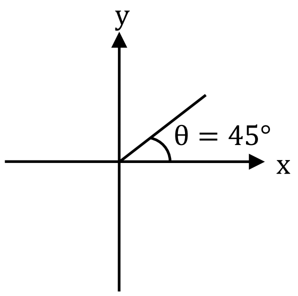
$10y(3) + y(4) = 10 \times 3 + 1 = 31$

45. A half wavelength dipole is kept in the x-y plane and oriented along 45° from the x-axis. Determine the direction of null in the radiation pattern for $0 \le \phi \le \pi$. Here the angle $\theta(0 \le \theta \le \pi)$ is measured from the z-axis, and the angle $\phi(0 \le \phi \le 2\pi)$ is measured from the x-axis in the x-y plane.

(A) θ=90°, ϕ=45°
(B) θ=45°, ϕ=90°
(C) θ=90°, ϕ=135°
(D) θ=45°, ϕ=135°

[Ans. A]

As the antenna is placed in xy − plane which is horizontal plane i. e $\theta = \frac{\pi}{2}$



$\therefore$ Antenna in $\theta = \dfrac{\pi}{2}$

As there is no field along antenna i.e null along antenna, $\phi = 45°$as $0 \le \phi \le \pi$ given

$\therefore$ For the given antenna null is at $\theta = 90, \phi = 45°$

46. Let $x(t)$ be a continuous time periodic signal with fundamental period $T = 1$ seconds, Let $\{a_k\}$ be the complex Fourier series coefficients of $x(t)$, where k is integer valued. Consider the following statement about $x(3t)$:

I. The complex Fourier series coefficients of $x\,(3t)$ are $\{a_k\}$ where k is integer valued

II. The complex Fourier series coefficients of $x\,(3t)$ are $\{3a_k\}$where k is integer valued

III. The fundamental angular frequency of $x\,(3t)$ is 6π rad/s

For the three statements above, which one of the following is correct?

(A) only II and III are true
(B) only I and III are true
(C) only III is true
(D) only I is true

[Ans. B]

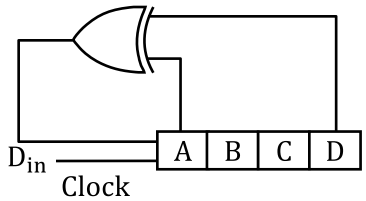
$x(t) \to a_k, \omega_o = 2\pi$

$x(at) \to a_k, a\omega_o$

$x(3t) \to a_k, 3\omega_o = 6\pi$

so I and III are true

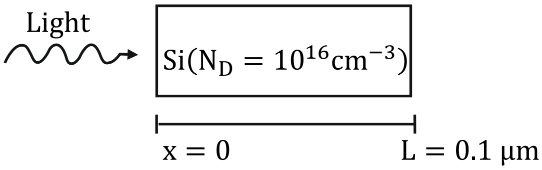
47. A 4-bit shift register circuit configured for right-shift operation is $D_{in} \to A, A \to B, B \to C, C \to D$, is shown. If the present state of the shift register is ABCD = 1101, the number of clock cycles required to reach the state ABCD = 1111 is________



[Ans. *] Range: 10.0 to 10.0

Clk	A	B	C	D
0	1	1	0	1
1	0	1	1	0
2	0	0	1	1
3	1	0	0	1
4	0	1	0	0
5	0	0	1	0
6	0	0	0	1
7	1	0	0	0
8	1	1	0	0
9	1	1	1	0
10	1	1	1	1 $\Rightarrow$ Required state

48. As shown a uniformly doped silicon (Si) bar of length $L = 0.1\ \mu m$ with a donor concentration $N_D = 10^{16}\ cm^{-3}$ is illuminated at $x = 0$ such that electron and hole pairs are generated at the rate of $G_L = G_{L0}\left(1 - \frac{x}{L}\right)$, $0 \le x \le L$, Where $G_{L0} = 10^{17} cm^{-3} s^{-1}$. Hole lifetime is 10^{-4} s, electronic charge $q = 1.6 \times 1^{-19}C$, hole diffusion coefficient $D_P = 100\ cm^2/s$ and low level injection condition prevails. Assuming a linearly decaying steady state excess hole concentration that goes to 0 at $x = L$, the magnitude of the diffusion current density at $x = L/2$, in A/cm^2 is ________.



[Ans. *] Range: 15.9 to 16.1

$$J = -D_p q \frac{dp(x)}{dx}$$

$$P(x) = G_{Lo}\left(1 - \frac{x}{L}\right)\tau_{po} + P_{no} = 10^{17}\left(1 + \frac{x}{L}\right)10^{-4} + p_{no} = 10^{13}\left(1 - \frac{x}{L}\right) + p_{no}$$

$$J = (+)10^2 \times 1.6 \times 10^{-19} \times 10^{13}(+)\frac{1}{L} L = 10^{-5} m$$

$$= \frac{1.6 \times 10^{-4}}{10^{-5}}$$

$$= 1.6 \times 10 A/cm^2$$

$$= 16\ A/cm^2$$

49. Let $h[n]$ be the impulse response of a discrete-time linear time invariant (LTI) filter. The impulse response is given by

$$h(0) = \frac{1}{3}; h[1] = \frac{1}{3}; h[2] = \frac{1}{3}; \text{and } h[n[= 0 \text{ for } n < 0 \text{ and } n > 2$$

Let $H(\omega)$ be the Discrete-Time Fourier transform (DTFT) of $h[n]$, where ω is the normalized angular frequency in radians. Given that $H(\omega_0) = 0$ and $0 < \omega_0 < \pi$, the value of ω_0 (in radians) is equal to _____.

[Ans. *] Range: 2.05 to 2.15

$$h(n) = \left[\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right]$$

$$H(e^{j\omega}) = \frac{1}{3} + \frac{1}{3}e^{-j\omega} + \frac{1}{3}e^{-j2\omega} = \frac{1}{3}e^{-j\omega}\left[e^{j\omega} + e^{-j\omega}\right] + \frac{1}{3}e^{-j\omega}$$

$$H(e^{j\omega}) = \frac{2}{3}e^{-j\omega}\cos\omega \frac{1}{3}e^{-j\omega}$$

$$= \frac{1}{3}e^{-j\omega}[1 + 2\cos\omega]$$

$H(e^{j\omega}) = 0$ when

$$\Rightarrow 1 + 2\cos\omega = 0$$

$$\Rightarrow \cos\omega = \frac{-1}{2}$$

$$\Rightarrow \omega = \cos^{-1}\left(-\frac{1}{2}\right) = 120° = \frac{2\pi}{3} = 2.094 \text{ rad}$$

50. Which one of the following is the general solution of the first order differential equation

$$\frac{dy}{dx} = (x + y - 1)^2$$

Where x, y are real?

(A) $y = 1 + x + \tan^{-1}(x + c)$ where c is a constant
(B) $y = 1 + x + \tan(x + c)$, where c is a constant
(C) $y = 1 - x + \tan^{-1}(x + c)$, where c is a constant
(D) $y = 1 - x + \tan(x + c)$, where c is a constant

[Ans. D]

$$\frac{dy}{dx} = (x + y - 1)^2 \rightarrow (1)$$

Put $x + y - 1 = t$

$$1 + \frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dy}{dx} = \frac{dt}{dx} - 1$$

$$(1) \Rightarrow \frac{dt}{dx} - 1 = t^2$$

$$\Rightarrow \frac{dt}{dx} = 1 + t^2$$

$$\Rightarrow \frac{dt}{1 + t^2} = dx$$

Integrating both sides

$$\int \frac{dt}{1 + t^2} = \int dx$$

$$\Rightarrow \tan^{-1} t = x + c$$

$$\therefore \tan^{-1}(x + y - 1) = x + c$$

(or) $x + y - 1 = \tan(x + c)$

(or) $y = 1 - x + \tan(x + c)$ is the solution

51. The expression for an electric field in free space is $E = E_0(\hat{x} + \hat{y} + j2\hat{z})\, e^{-j(\omega t - kx + ky)}$, where x,y,z represent the spatial coordinates, t represents time, and ω, k are constants. This electric field

(A) does not represent a plane wave.
(B) represents a circularly polarized plane wave propagating normal to the z-axis.
(C) represents an elliptically polarized plane wave propagating along the x-y plane.
(D) represents a linearly polarized plane wave.

[Ans. C]

$$E = E_0(\hat{x} + \hat{y} + j2\hat{z})e^{-j(\omega t - kx + ky)}$$

$$e^{-jkr} = e^{-j(-kx + ky)}$$

$$\therefore kr = k(-x + y)$$

Propagation vector $\hat{a}_p = \dfrac{\nabla(kr)}{|\nabla(kr)|}$

$$\nabla(kr) = k(-\hat{x} + \hat{y})$$

$$|\nabla(kr)| = k\sqrt{2}$$

$$\hat{a}_p = \frac{\nabla(kr)}{|\nabla(kr)|} = \frac{-\hat{x}+\hat{y}}{\sqrt{2}}$$

For plane wave $\hat{a}_p \cdot \hat{E} = 0$

$$\hat{a}_p\hat{E} = \left[\frac{-\hat{x}+\hat{y}}{\sqrt{2}}\right] \cdot E_o[\hat{x}+\hat{y}+j2\hat{z}]$$

$$= -\frac{E_o}{\sqrt{2}} + \frac{E_o}{\sqrt{2}} + j0$$

$\hat{a}_p \cdot \hat{E} = 0 \therefore$ Given is a plane wave .

As $E = E_o(\hat{x}+\hat{y}+j2\hat{z})e^{-j(\omega t - kx + ky)}$

For the given wave, plane of incidence is xy-plane.

E in xy plane is parallel polarized

and along z is perpendicular polarized

$E\| = |E|_{xy} = \sqrt{1+1} = \sqrt{2}$

$E\| = |E|_z = \sqrt{2^2} = 2$

$|E_T| = |E\|| + |E_{\perp r}|$

$= \sqrt{2} + 2$

$|E\|| \neq |E_{\perp r}|$and phase difference is $90°$; i.e. given is a

Elliptically polarized plane wave

52. Which one of the following options correctly describes the locations of the roots of the equation $s^4 + s^2 + 1 = 0$ on the complex plane?

(A) Four left half plane (LHP) roots

(B) One right half plane (RHP) root, one LHP root and two roots on the imaginary axis

(C) Two RHP roots and two LHP roots

(D) All four roots are on the imaginary axis

[Ans. C]

CE $S^4 + S^2 + 1 = 0$

$$\begin{array}{cc|cccl}
 & S^4 & IS^4 & 1S^2 & 1S^0 & \\
 & S^3 & \cancel{0}\ 4 & \cancel{0}\ 2 & \cancel{0} & \text{IROZ} \\
 & S^2 & 0.5 & 1 & & \\
1 & S^1 & \frac{-3}{0.5} = -6 & & & \\
2 & S^0 & 1 & & &
\end{array}$$

$\Rightarrow$ 2 sign changes and 1 ROZ

$\Rightarrow$ 2 poles in right half of S-plane

And symmetrical poles in the LHS-plane.

53. The following FIVE instructions were executed on an 8085 microprocessor.

MVI A, 33H

MVI B, 78H

ADD B

CMA

ANI 32H

The Accumulator value immediately after the execution of the fifth instruction is

(A) 00H
(B) 10H
(C) 11H
(D) 32H

[Ans. B]

MVI A, 33H: $A \leftarrow 33_H$

MVI B, 78H: $B \leftarrow 78_H$

ADD B:

$$\begin{array}{rcc} & 3 & 3 \\ + & 7 & 8 \\ \hline & \multicolumn{2}{c}{AB_H} \end{array}$$

$A \leftarrow ABH$

CMA: $A = 1010\ 1011$

$\overline{A} = 0101\ 0100$

$A \leftarrow 54_H$

ANI 32H:

$$\begin{array}{cccccccc} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array}$$

10_H

$A \leftarrow 10_H$

54. In binary frequency shift keying (FSK), the given signal wave forms are

$u_{0(t)} = 5\ \cos(20000\pi t)\,; 0 \le t \le T$, and

$u_1(t) = 5\cos(22000\pi t)\,; 0 \le t \le T$,

where T is the bit-duration interval and t is in seconds. Both $u_0(t)$ and $u_1(t)$ are zero outside the interval $0 \le t \le T$. With a matched filter (correlator) based receiver, the smallest positive value of T (in milliseconds) required to have $u_0(t)$ and $u_1(t)$ uncorrelated is

(A) 0.25 ms
(B) 0.5 ms
(C) 0.75 ms
(D) 1.0 ms

[Ans. B]

Method 1

$$f_1 = \frac{20000\pi}{2\pi} = 10k$$

$$f_2 = \frac{22000\pi}{2\pi} = 11k$$

$$f_2 - f_1 = \frac{1}{2Tb}\text{[Condition for un} - \text{correlation in coherent FSK]}$$

$$T_b = \frac{1}{2(f_2 - f_1)}$$

$$T_b = 0.5\text{ ms}$$

Method 2

$$\int_0^T u_0\,(t)u_1(f) = 0\ \ \text{[If two signals are un} - \text{correlated]}$$

$$\int_0^T \frac{1}{2}[25\cos(42000\pi t) + 25\cos\pi t)]\,dt = 0$$

$$\underbrace{\frac{25}{2}\frac{\sin(42000\pi T)}{42000\pi}}_{1} + \underbrace{\frac{25}{2}\frac{\sin(2000\pi T)}{2000\pi}}_{2} = 0$$

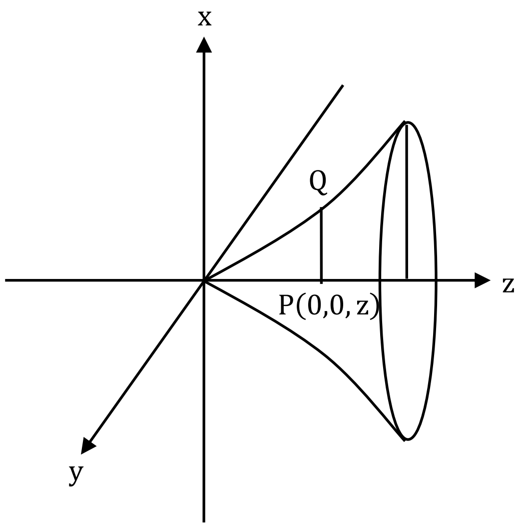
$\sin 2000\pi T = 0$ when $2000\pi T = \pi$

$$\Rightarrow T = \frac{1}{2000}$$

At $T = \frac{1}{2000}$, $1 = 2 = 0$

$\therefore T = 0.5$ ms

55. A three dimensional region R of finite volume is described by
$x^2 + y^2 \le z^3; 0 \le z \le 1,$
Where x, y, z are real. The volume of R (up to two decimal places) is_______
[Ans. *] Range: 0.70 to 0.85



Let $x^2 + y^2 = t^2$
$t^2 = z^3$
Here revolution is about z axis

$$\text{Volume of region R} = \int_0^1 \pi(PQ)^2 dz$$

Here PQ is radius of circle at some Z, which is given by

$PQ = \sqrt{x^2 + y^2}$

$(PQ)^2 = x^2 + y^2 = z^3$

$$\text{So, Volume of region R} = \int_0^1 \pi t^2 dz = \pi Z^3 dz = \left.\frac{\pi z^4}{4}\right|_0^1$$

$$= \frac{\pi}{4} = 0.7853$$