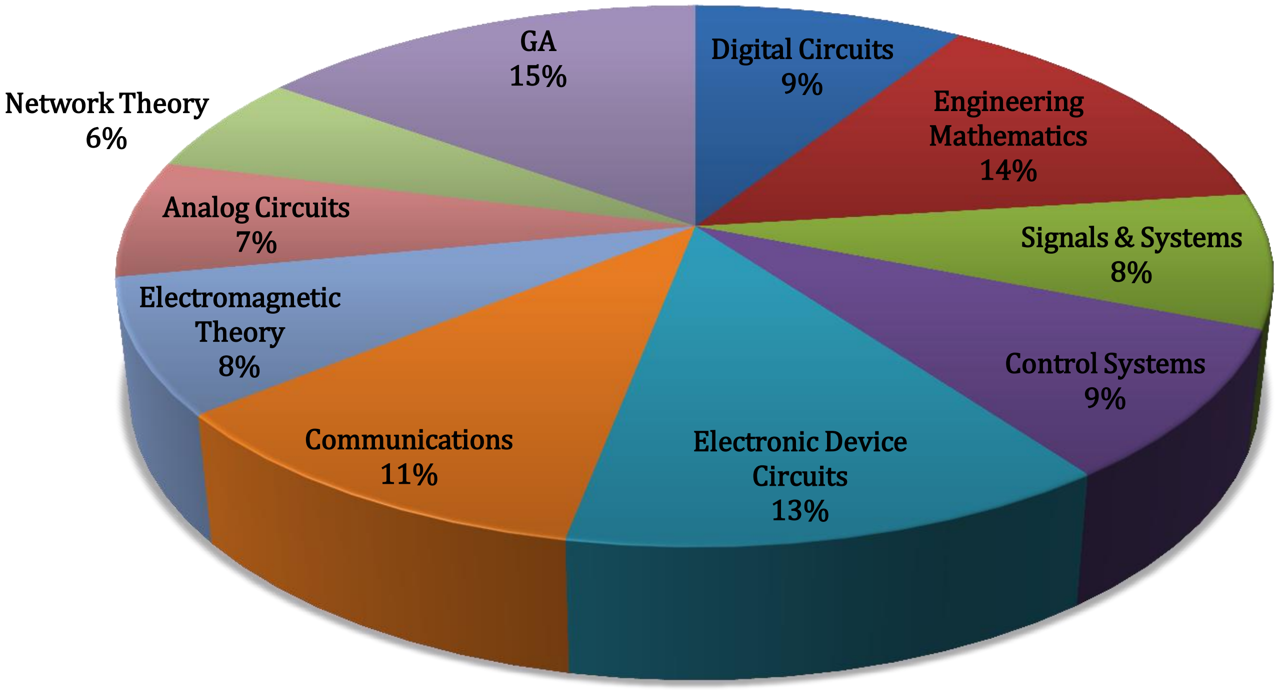
ANALYSIS OF GATE 2017
Electronics and Communication Engineering



ECE ANALYSIS-2017_5-Feb_Afternoon

SUBJECT	Ques. No.	Topics Asked in Paper(Memory Based)	Level of Toughness	Total Marks
Engineering Mathematics	1 Marks:4 2 Marks:5	Complex variable (Residue, Complex integral), Calculus (vector calculus)	Easy	14
Network Theory	1 Marks:2 2 Marks:2	Steady stat analysis, Transient, R-L-C circuit	Tough	6
Signals & Systems	1 Marks:2 2 Marks:3	Filter, Sampling theorem, LTI system, Parallel connection	Tough	8
Control Systems	1 Marks:3 2 Marks:3	State space analysis, Block diagram, Time domain analysis, Nyquist plot	Medium	9
Analog Circuits	1 Marks:3 2 Marks:2	Op-amp, Diode circuit, BJT	Medium	7
Digital Circuits	1 Marks:3 2 Marks:3	Adder, FSM, Mux	Medium	9
Communications	1 Marks:3 2 Marks:4	Channel capacity, PCM, Sampling	Tough	11
Electronic Device Circuits	1 Marks:3 2 Marks:5	MOS CAP, MOSFET, BJT, PN	Easy	13
Electromagnetic Theory	1 Marks:2 2 Marks:3	Oblique insistence, Electrostatic, Waveguides	Medium	8
General Aptitude	1 Marks:5 2 Marks:5	Passage, Grammar, Time and work, Blood relation, Direction, Number system	Easy	15
Total	65			100
Faculty Feedback	80% of the problems are based on previous year problem. Few question of Mechanical 2017 exam is matching with EC 2017 paper and few similar to previous years Mathematics questions. Overall question paper was easy.			

GATE 2017 Examination

Electronics and Communication Engineering

Test Date: 05/02/2017

Test Time: 2:00 AM to 5:00 PM

Subject Name: Electronics and Communication Engineering

Section: General Aptitude

1. A rule states that in order to drink beer, one must be over 18 years old. In a bar, there are 4 people. P is 16 years old, Q is 25 years old, R is drinking milkshake and S is drinking a beer. What must be checked to ensure that the rule is being followed?
(A) only P's drink
(B) Only P's drink and S's age
(C) only S's age
(D) only P's age drink. Q's drink and S's age
[Ans. B]
From the given data is P's age is 16 years it is under 18 years of age so, drink is need to check and 'S' is drinking a beer so, his age is more than 18 years (or) not also need to check from the rules given above.

2. The ninth and the tenth of this month are Monday and Tuesday ________
(A) figuratively
(B) retrospectively
(C) respectively
(D) rightfully
[Ans. C]
'Respectively' means in the same order as the people or things already mentioned.

3. 500 students are taking one or more courses out of chemistry, physics and Mathematics. Registration records indicate course enrolment as follows: chemistry (329), physics (186), Mathematics (295), chemistry and physics (83), chemistry and Mathematics (217), and physics and Mathematics (63), How many students are taking all 3 subjects
(A) 37
(B) 43
(C) 47
(D) 53
[Ans. D]
Chemistry = C, physics = P and
Mathematics = M
$n(CUPUM) = 500, n(C) = 329, n(P) = 186$
$n(M) = 295, n(C \cap P) = 83, n(C \cap M) = 217$
and $n(P \cap M) = 63$
$n\ (CUPUM) = n(C) + n(P) + n(M)(C \cap P) - n(C \cap M) - n(P \cap M) + n(P \cap C \cap M)$
$500 = 329 + 186 + 295 - 83 - 217 - 36 + n(C \cap P \cap M)$
$500 = 810 - 363 + n(C \cap P \cap M)$
$\therefore n(C \cap P \cap M) = 500 - 447 = 53$

4. It ________ to read this year's textbook ________ the last year's

(A) easier, than
(B) most easy, than
(C) easier, from
(D) easiest, from

[Ans. A]

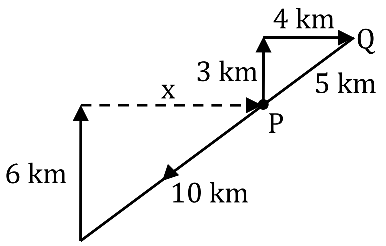
It is a comparative degree, so the right option is (A)

5. Fatima starts form point P, goes North for 3km, and then East for 4 km to reach point Q. She then turns to face point P and goes 15km in that direction. She then goes North for 6km. How far is she from point P, and in which direction should she go to reach point P?

(A) 8 km, East
(B) 12 km, North
(C) 6 km, East
(D) 10 km, North

[Ans. A]

From the given data, the following diagram is possible



$(\text{HYP})^2 = (\text{Opp . side})^2 + (\text{Adjacent side})^2$

$(10)^2 = (6)^2 + \text{x}2$

$\text{x}^2 = (10)^2 - (6)^2$

$\therefore \sqrt{100 - 36} = 8$ km

For Reading 'P' from the Reached position is 8 km towards East

6. "If you are looking for a history of India, or for an account of the rise and fall of the British Raj, or for the reason of the cleaving of the subcontinent into two mutually antagonistic parts and the effects this mutilation will have in the respective sections, and ultimately on Asia, you will not find it in these pages; for though I have spent a lifetime in the country. I lived too near the seat of events, and was too intimately associated with the actors, to get the perspective needed for the impartial recording of these matters".

Which of the following statements best reflects the author's opinion?

(A) An intimate association does not allow for the necessary perspective
(B) Matters are recorded with an impartial perspective
(C) An intimate association offers an impartial perspective
(D) Actors are typically associated with the impartial recording of matters.

[Ans. A]

7. The number of 3-digit numbers such that the digit 1 is never to the immediate right of 2 is

(A) 781
(B) 791
(C) 881
(D) 891

[Ans. C]

Total number of three digit numbers possible are $9 \times 10 \times 10 = 900$

Number of possibilities for digit '1' to be immediate right of digit '2' are

2	1	x

$1 \times 1 \times 10 = 10$

x	2	1

$9 \times 1 \times 1 = 9$

$= 19$

So, number of possibilities such that the digit '1' is never to the immediate right of '2' are $900 - 19 = 881$

8. Each of P, Q, R, S, W, X, Y and Z has been married at most once. X and Y married and have two children P and Q .Z is the grandfather of the daughter S of P. Further, Z and W are married and are parents of R. Which one of the following must necessarily be FALSE?

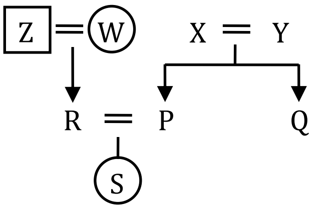
(A) X is the mother in law of R

(B) P and R not married to each other

(C) P is a son of X and Y

(D) Q cannot be married to R

[Ans. B]



Where,

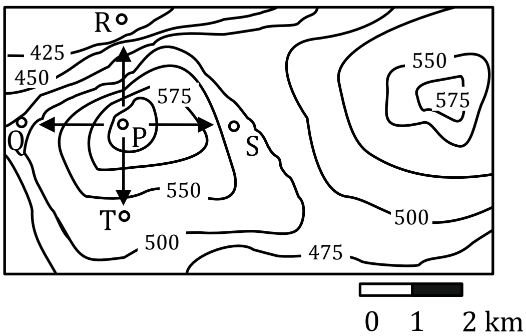
□ → Male

○ → Female

═ → Sing for marriage

From diagram P and R are married to each other

9. A contour line joins locations having the same height above the mean sea level. The following is a contour plot of a geographical region.



Contour lines are shown at 25m intervals in this spot. Which of the following is the steepest path leaving from P?

(A) P to Q

(B) P to R

(C) P to S

(D) P to T

[Ans. B]

Form the given contour lines and Locations

The path from P to Q = 575 to 500 = 75 m deep

The path from P to R = 575 to 425 = 150 m deep
The path from P to S = 575 to 525 = 25 m deep
The path from P to T = 575 to 525 = 25 m deep
Among all of this paths P to R is the steepest path

10. 1200 men and 500 women can build a bridge in 2 weeks. 900 men and 250 women will take 3 weeks to build the same bridge. How many men will be needed to build the bridge in one week?

(A) 3000 (C) 3600
(B) 3300 (D) 3900

[Ans. C]

Let a man can build the bridge = x weeks
A woman can build the bridge = y weeks
From the given data,

$\frac{1200}{x} + \frac{500}{y} = \frac{1}{2} \ldots ①$

$\frac{900}{x} + \frac{250}{y} = \frac{1}{3} \ldots ②$

By solving equation ① and ②, x = 3600
∴ A man can build the bridge in 3600 weeks

Section: Technical

1. The smaller angle (in degrees) between the planes $x + y + z = 1$ and $2x - y + 2z = 0$ is ______

[Ans. *] Range: 54.0 to 55.0

Angle between two planes
$a_1x + b_1y + c_1z = d_1$ and $a_2x + b_2y + c_2z = d_2$ is given by

$$\cos\theta = \frac{|a_1a_2 + b_1b_2 + c_1c_2|}{\sqrt{a_1^2 + b_1^2 + c_1^2}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

Now, the angle between $x + y + z = 1$ and $2x - y + 2z = 0$ is

$$\cos\theta = \frac{2 - 1 + 2}{\sqrt{1 + 1 + 1}\sqrt{4 + 1 + 4}} = \frac{3}{3\sqrt{3}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) = 54.73°$$

2. A sinusoidal message signal is converted to a PCM signal using a uniform quantizer. The required signal to quantization noise ratio (SQNR) at the output of the quantizer is 40dB. The minimum number of bits per sample needed to achieve the desired SQNR is______

[Ans. *] Range: 7 to 7

The signal to Noise ratio in a uniform Quantizer is

$SNR = [\,1.8 + 6n\,] \geq 40$ dB

$6n \geq 40 - 1.8$

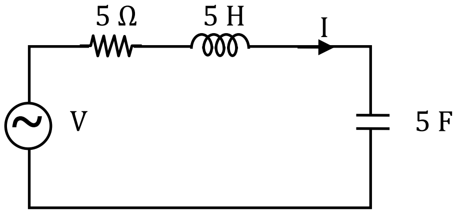
≥ 38.2

$n \geq \frac{38.2}{6}$

≥ 6.36

So $n_{min} = \text{(integer)} = 7$

3. In the circuit shown, V is a sinusoidal voltage source. The current I is in phase with voltage V. The ratio $\frac{\text{Amplitude of voltage across the capcitor}}{\text{Amplitude ofvoltage acrros the resistor}}$ is ________



[Ans. *] Range: 0.19 to 0.21

Given that V and I are in phase

⇒ Circuit is at resonance

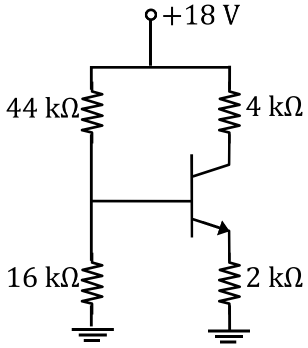
$\Rightarrow V_C = QV\angle - 90°$

$V_R = V$

$$\rightarrow \frac{|V_C|}{|V_R|} = \frac{QV}{V} = Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$$= \frac{1}{5}\sqrt{\frac{5}{5}} = 0.2$$

4. Consider the circuit shown in figure. Assume base to emitter voltage $V_{BE} = 0.8V$ and common base current gain (α) of transistor is unity.



The value of the collectors to emitter voltage V_{CE} (in volt) is ________

[Ans. *] Range: 5.5 to 6.5

$\alpha = 1 \Rightarrow \beta \approx \infty \Rightarrow I_B = 0$

$$V_B = 18 \times \frac{16}{16 + 44} = 18 \times \frac{16}{60} = 4.8 \text{ V}$$

$V_E = 4.8 - 0.8 = 4V$

$$I_E = \frac{4}{2k} = 2 \text{ mA}$$

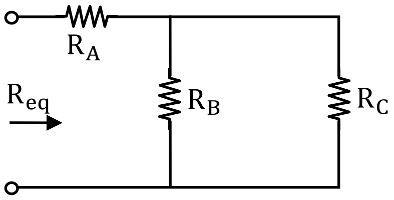
$I_C = I_E = 2 \text{ mA}$

$V_C = 18 - (4 \text{ k} \times 2 \text{ m}) = 10 \text{ V}$

$V_{CE} = 10 - 4 = 6 \text{ V}$

5. A connection is made consisting of resistance A is series with a parallel combination of resistance B and C. Three resistors of value 10Ω, 5 Ω, 2 Ω are provided. Consider all possible permutations of the given resistors into the positions A, B, C and identify the configuration with maximum possible overall resistance; and also the ones with minimum possible overall resistance. The ratio of maximum to minimum values of the resistances (up to second decimal place) is

[Ans. *] Range: 2.12 to 2.16



Given that, R_A or R_B or $R_C = 10\Omega$

R_A or R_B or $R_C = 5\Omega$

R_A or R_B or $R_C = 2\Omega$

For R_{eq} maximum:

The required combination is

$R_A = 10\Omega$ and $R_B = 5\Omega$ or 2Ω

And $R_C = 2\Omega$ or 5Ω

So, $R_{eq} = R_A + (R_B \| R_C)$

$$= 10 + (2\|5) = \frac{80}{7} = 11.4285\ \Omega$$

$= $ Req max.

For R_{eq} minimum:

The required combination is

$R_A = 2\Omega$ and $R_B = 10\Omega$ or 5Ω

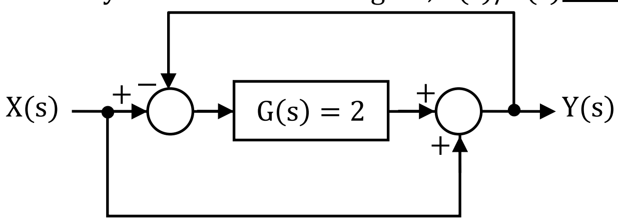
So, $R_{eq} = R_A + (R_B \| R_C) = 2 + (10\|5)$

$$= \frac{16}{3}\Omega = 5.33\ \Omega$$

$= $ Req min

Hence, $\frac{R_{eq}\text{max}}{R_{eq}\text{min}} = \frac{11.4285}{5.33} = 2.143$

6. For the system shown in the figure, $Y(s)/X(s)$________



[Ans. *] Range: 0.95 to 1.05

$$\frac{Y(s)}{X(s)} = \frac{2+1}{1+2} = 1$$

7. The general solution of the differential equation

$$\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$$

in terms of arbitrary constants K_1 and K_2 is

(A) $K_1e^{(-1+\sqrt{6})x}+K_2e^{(-1-\sqrt{6})x}$

(B) $K_1e^{(-1+\sqrt{8})x}+K_2e^{(-1-\sqrt{8})x}$

(C) $K_1e^{(-2+\sqrt{6})x}+K_2e^{(-2-\sqrt{6})x}$

(D) $K_1e^{(-2+\sqrt{8})x}+K_2e^{(-2-\sqrt{8})x}$

[Ans. A]

Given, $\frac{d^2y}{dx^2}+2\frac{dy}{dx}-5y=0$

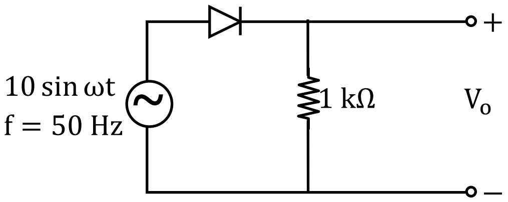
Auxiliary equation is $D^2+2D-5=0$

Roots are $-1\pm\sqrt{6}$

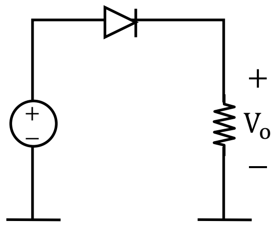
∴ The general Solution is

$y=K_1e^{(-1+\sqrt{6})x}+K_2e^{(-1-\sqrt{6})x}$

8. The output V_0 of the diode circuit shown in figure is connected to an averaging DC voltmeter. The reading on the DC voltmeter in volts, neglecting the voltage drop across the diode, is________



[Ans. *] Range: 3.15 to 3.21



$V_{o,Avg}=\frac{10}{\pi}=3.18\text{ V}$

9. The input x(t) and the output y(t) of a continuous time system are related as

$y(t)=\int_{t-T}^{t}x(u)du$. The system is

(A) Linear and time variant

(B) linear and time invariant

(C) non linear and time variant

(D) nonlinear and time invariant

[Ans. B]

$$y(t)=\int_{t-T}^{t}x(u)du$$

The system is linear, it satisfied both superposition and scaling property.

$$y(t)=\int_{t-T}^{t}x(u-\tau)du$$

$u-\tau=\lambda$

$du=d\lambda$

$$y_1(t) = \int_{t-T-\tau}^{t-\tau} x(\lambda)d\lambda \ldots\ldots\ldots.. \text{①}$$

$$y(t-\tau) = \int_{t-T-\tau}^{t-\tau} x(u)du \ldots\ldots\ldots. \text{②}$$

$y_1(t) = y(t-\tau)$

So, Time invarient

10. Which of the following statement is incorrect?
    (A) Lead compensator is used to reduce the settling time.
    (B) Lag compensator is used to reduce the steady state error.
    (C) Lead compensator may increase the order of a system
    (D) Lag compensator always stabilizes an unstable system.

[Ans. D]

Lag compensator reduces the steady state error but it cannot stabilizes an unstable system.

11. Consider an n-channel MOSFET having width W, length L, electron mobility in the channel μ_n and oxide capacitance per unit area C_{ox}. If gate-to-source voltage $V_{GS} = 0.7V$, drain-to-source voltage $V_{DS} = 0.1V$, $(\mu_n C_{ox}) = 100\ \mu A/V^2$, threshold voltage $V_{TH} = 0.3V$ and $(W/L) = 50$, then the transconductance g_m (in mA/V) is

[Ans. *]Range: 0.45 to 0.55

$V_{GS} = 0.7$

$V_{DS} = 0.1; V_{TH} = 0.3V$

$V_{DS} < V_{GS} = V_{TH}$

$0.1 < 0.7 - 0.3$

$0.1 < 0.4 \Rightarrow \text{TRIODE}$

$$I_D = \mu_n C_{ox} \frac{W}{L}\left[(V_{GS} - V_T)V_{DS} - \frac{1}{2}V_{DS}^2\right]$$

$$g_m = \frac{\partial I_G}{\partial V_{GS}} = \mu_n C_{ox}\frac{W}{L}[V_{DS}]$$

$= 100 \times 10^{-6} \times 50 \times 0.1$

$= 0.5 \times 10^{-3}$

$= 0.5\ mA/V$

12. An LTI system with unit sample response $h(n) = 5\delta[n] - 7\delta[n-1] + 7\delta[n-3] - 5\delta[n-4]$ is a
    (A) low pass filter
    (B) high pass filter
    (C) band pass filter
    (D) band stop filter

[Ans. C]

$h(n) = 5\delta(n) - 7\delta(n-1) + 7\delta(n-3) - 5\delta(n-4)$

In frequency domain

$H(e^{j\omega}) = 5 - 7e^{-j\omega} + 7e^{-j3\omega} - 5e^{-j4\omega}$

$\Rightarrow H(e^{j\omega}) = 5e^{-j2\omega}(e^{j2\omega} - e^{-j2\omega}) - 7e^{-j2\omega}(e^{+j\omega} - e^{-j\omega})$

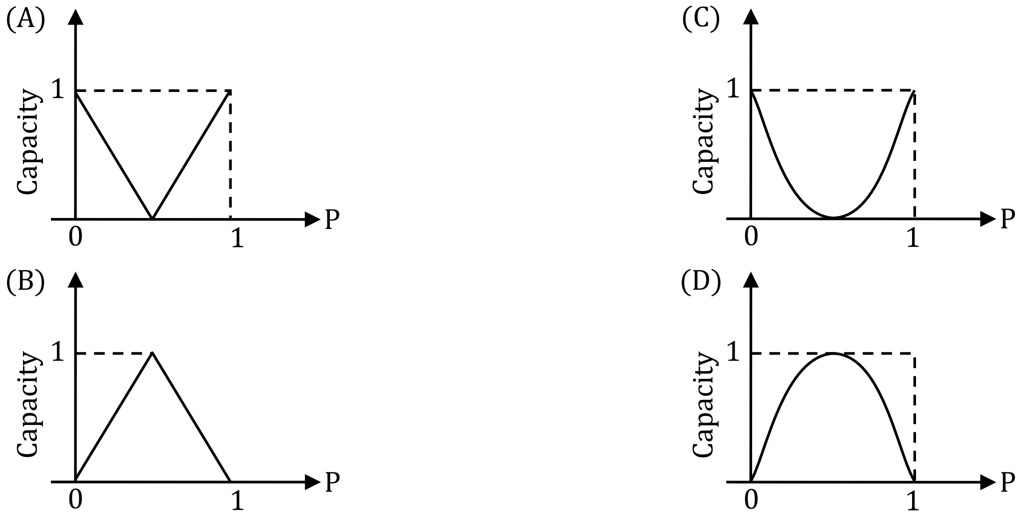
$= 5e^{-j2\omega}[2j\sin(2\omega)] - 7e^{-j2\omega}[2j\sin\omega]$

$= e^{-j2\omega}[10\, j\sin 2\omega - 14\, j\sin\omega]$

ω	$H(e^{j\omega})$	$\lvert H(e^{j\omega})\rvert$
0	0	0
$\pi/2$	14j	14
π	0	0

It is a Band pass filter.

13. Which one of the following graphs shows the Shannon capacity (channel capacity) in bits of a memory-less binary symmetric channel with crossover probability p?



[Ans. C]

The channel capacity of a BSC channel is

$C = 1 + P\log_2 P + (1-P)\log_2(1-P)$

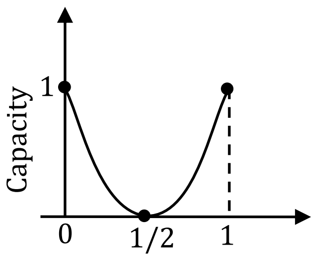
When,

$P = 0 \Rightarrow C = 1$

$P = 1/2 \Rightarrow C = 0$

$P = 1 \Rightarrow C = 1$

Channel capacity of a BSC



14. In a DRAM,

(A) Periodic refreshing is not required

(B) information is stored in a capacitor

(C) information is stored in a latch

(D) both read and write operations can be performed simultaneously

[Ans. B]

In DRAM

(i) Periodic refreshing is required
(ii) Information is stored in a capacitor
(iii) Information is not stored in a latch
(iv) Both Read and Write operations cannot be performed simultaneously

15. A two wire transmission line terminates in a television set. The VSWR measured on the line is 5.8. The percentage of power that is reflected from the television set is_______

[Ans. *] Range: 48.0 to 51.0

$S = 5.8$

$$|\Gamma| = \frac{S-1}{S+1} = \frac{4.8}{6.8} = 0.705$$

$\%$ of reflected power $= |\Gamma^2| \times 100 = (0.705)^2 \times 100 = 49.82\%$

16. An n-channel enhancement mode MOSFET is biased at $V_{GS} > V_{TH}$ and $V_{DS} > (V_{GS} - V_{TH})$, where V_{GS} is the gate to source voltage, V_{DS} is the drain to source voltage and V_{TH} is the threshold voltage. Considering channel length modulation effect to be significant, the MOSFET behaves as a _____

(A) Voltage source with zero output impedance
(B) Voltage source with non-zero output impedance
(C) Current source with finite output impedance
(D) Current source with infinite output impedance

[Ans. C]

If the effect of channel length modulation is considered then the output resistance is finite value.

17. The rank of the matrix

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \text{ is } \_\_\_\_\_\_$$

[Ans. *] Range: 4 to 4

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \to R_4 + R_1$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \rightarrow R_4 + R_2$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_4 \rightarrow R_4 + R_3$

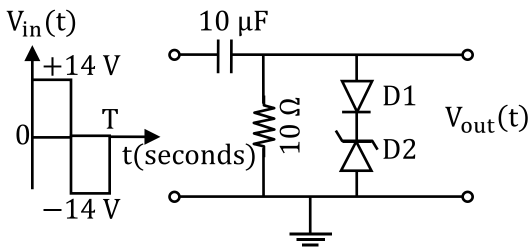
$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$R_5 \rightarrow R_5 + R_4$

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore \rho(A) = 4$

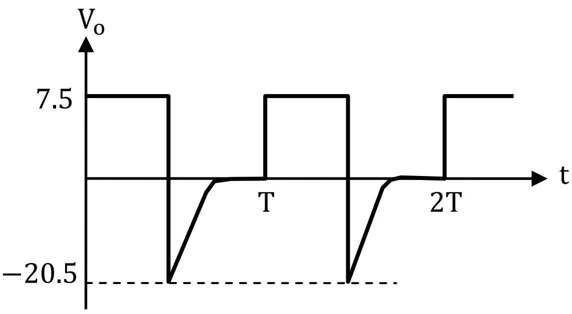
18. In the figure, D1 is a real silicon pn junction diode with a drop of 0.7V under forward bias condition and D2 is a zener diode with breakdown voltage of -6.8V. The input $V_{in}(t)$ is a periodic square wave of period T, whose one period is shown in the figure.



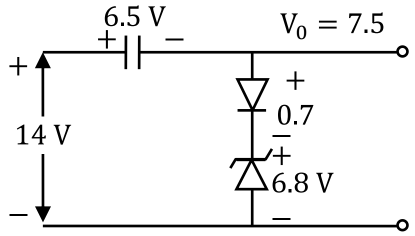
Assuming $10\,\tau \ll T$. Where τ is the time constant of the circuit, the maximum and minimum values of the output waveform are respectively?

(A) 7.5V and -20.5V
(B) 6.1V and -21.9V
(C) 7.5 V and -21.2V
(D) 6.1V and -22.6V

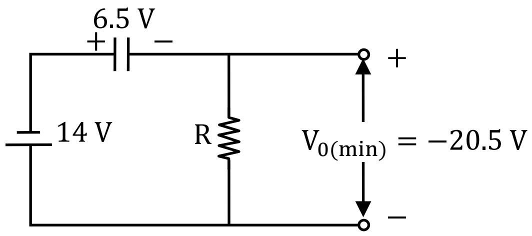
[Ans. A]



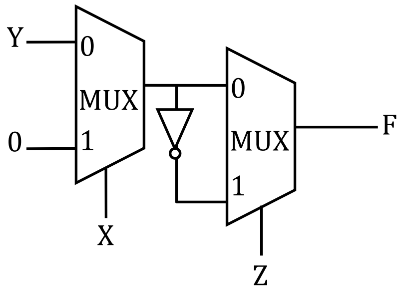
+ve cycle:



− ve cycle:



19. Consider the circuit shown in figure.



The Boolean expression F implemented by the circuit is

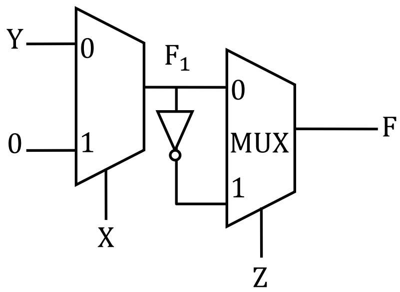
(A) $\bar{X}\,\bar{Y}\bar{Z} + XY + \bar{Y}Z$

(B) $\bar{X}Y\bar{Z} + XZ + \bar{Y}Z$

(C) $\bar{X}Y\bar{Z} + XY + \bar{Y}Z$

(D) $\overline{XY}\bar{Z} + XZ + \bar{Y}Z$

[Ans. B]



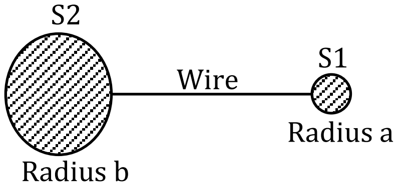
$F_1 = \bar{X}Y + X.0 = \bar{X}Y$

$F = \bar{Z}F_1 + Z\bar{F}_1$

$= \bar{Z}(\bar{X}Y) + Z(X + \bar{Y})$

$= \bar{X}Y\bar{Z} + XZ + \bar{Y}Z$

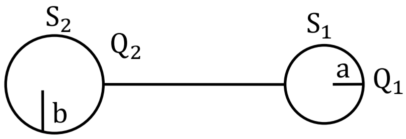
20. Two conducting spheres S1 and S2 of radii a and b (b>a) respectively, are placed far apart and connected by a long, thin conducting wire, as shown in the figure.



For some charge placed on this structure, the potential and surface electric field on S1 are V_a and E_a, and that on S2 are V_b and E_b respectively. Then, which of the following is CORRECT?

(A) $V_a = V_b$ and $E_a < E_b$
(B) $V_a > V_b$ and $E_a > E_b$
(C) $V_a = V_b$ and $E_a > E_b$
(D) $V_a > V_b$ and $E_a = E_b$

[Ans. C]



When the two spheres are connected by a conducting wire, charge will flow from one to another until their potentials are equal.

$V_a = V_b$

$$\frac{1}{4\pi\epsilon}\frac{Q_1}{a} = \frac{1}{4\pi\epsilon}\frac{Q_2}{b}$$

$$\frac{Q_1}{Q_2} = \frac{a}{b}$$

$$E_a = \frac{1}{4\pi\epsilon}\frac{Q_1}{a^2}$$

$$E_b = \frac{1}{4\pi\epsilon}\frac{Q_2}{b^2}$$

$$\therefore \frac{E_a}{E_b} = \frac{Q_1}{Q_2}\frac{b^2}{a^2}$$

$$\frac{E_a}{E_b} = \frac{b}{a}$$

So, $V_a = V_b$

And $E_a > E_b$

21. Consider the random process $X(t) = U + Vt$.
Where U is a zero mean Gaussian random variable and V is a random variable uniformly distributed between 0 and 2. Assume that U and V are statistically independent. The mean value of the random process at $t = 2$ is________.

[Ans. *]Range: 2.0 to 2.0

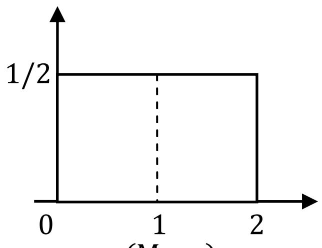
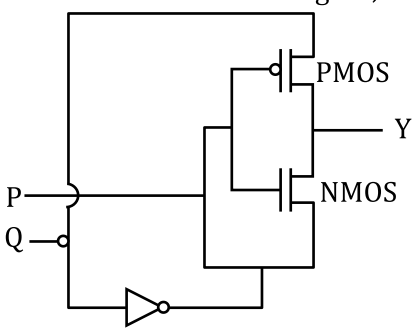


Figure: pdf of V

$X(t) = U + VT$
$E[U] = 0, E[V] = 1$
$E[X(t)] = E[U + Vt]$
$= E[U] + E[V]t$
$= 0 + 1 \times t = t$
$E[X(t)]_{at\ t=2} = 2$

22. For the circuit shown in figure, P and Q are the inputs and Y is the output.



The logic implemented by the circuit is
(A) XNOR
(B) XOR
(C) NOR
(D) OR

[Ans. B]

If $P = \text{high} \Rightarrow \left.\begin{matrix}\text{PMOS is OFF}\\ \text{NMOS is ON}\end{matrix}\right\} \Rightarrow y = \bar{Q}$

If $P = \text{low} \Rightarrow \left.\begin{matrix}\text{PMOS is ON}\\ \text{NMOS is OFF}\end{matrix}\right\} \Rightarrow y = Q$

P	Q	Y
0	0	0
0	1	1
1	0	1
1	1	0

Ex-OR Gate

23. Consider the state space realization

$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 0\\ 0 & -9\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix} + \begin{bmatrix}0\\ 45\end{bmatrix}u(t)$, With the initial condition $\begin{bmatrix}x_1(0)\\ x_2(0)\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$;

Where u(t)denotes the unit step function. The value of $\lim_{t\to\infty}\left|\sqrt{x_1^2(t) + x_2^2(t)}\right|$ is______

[Ans. *]Range: 4.99 to 5.01

$x(t) = ZIR + ZSR$

$ZIR = e^{At}x(0) = \begin{bmatrix}0\\0\end{bmatrix}$

$ZSR = L^{-1}[\phi(s)BU(S)]$

$(SI - A) = \begin{bmatrix}s & 0\\0 & s+9\end{bmatrix}$, Adj $(SI - A)\begin{bmatrix}s+9 & 0\\0 & s\end{bmatrix}$

$$\phi(s) = (SI - A)^{-1} = \frac{\text{Adj }(SI - A)}{|SI - A|} = \begin{bmatrix}\frac{1}{s} & 0\\0 & \frac{1}{s+9}\end{bmatrix}$$

$$ZSR = L^{-1}\left[\begin{bmatrix}\frac{1}{s} & 0\\0 & \frac{1}{s+9}\end{bmatrix}\begin{bmatrix}0\\45\end{bmatrix}\begin{bmatrix}\frac{1}{s}\end{bmatrix}\right] = L^{-1}\begin{bmatrix}0\\\frac{45}{s(s+9)}\end{bmatrix} = \begin{bmatrix}0\\5(1-e^{-9t})\end{bmatrix}$$

$x_1(t)0, x_2(t) = 5(1 - e^{-9t})$

$$\lim_{t\to\infty}\left|\sqrt{x_1^2(t) + x_2^2(t)}\right| = 5$$

24. The residues of function

$f(z) = \frac{1}{(z-4)(z+1)^3}$ are

(A) $\frac{-1}{27}$ and $\frac{-1}{125}$

(B) $\frac{1}{125}$ and $\frac{-1}{125}$

(C) $\frac{-1}{27}$ and $\frac{1}{5}$

(D) $\frac{1}{125}$ and $\frac{-1}{5}$

[Ans. B]

$$f(z) = \frac{1}{(z-4)(z+1)^3}$$

$$\operatorname*{Res}_{z=4} f(z) = \frac{1}{(4+1)^3} = \frac{1}{125}$$

$$\operatorname*{Res}_{z=-1} f(z) = \frac{1}{2!}\lim_{z\to-1}\frac{d^2}{dz^2}\left\{(z+1)^3\frac{1}{(z-4)(z+1)^3}\right\} = \frac{1}{2}\lim_{z\to-1}\left\{\frac{2}{(z-4)^3}\right\}$$

$$= \frac{1}{2}\left\{\frac{2}{-125}\right\}$$

$$= \frac{-1}{125}$$

25. An npn bipolar junction transistor (BJT) is operating in the active region. If the reverse bias across the base-collector junction is increased. Then

(A) the effective base width increases and common-emitter current gain increases
(B) the effective base width increases and common emitter current gain decreases
(C) the effective base width decreases and common-emitter current gain increases
(D) the effective base width decreases and common-emitter current gain decreases

[Ans. C]

If RB across the Base –collector junction increases

⇒ Effective Base width decreases

So re-combinations in Base decreases

So α increases, so β increases

26. Figure 1 shows a 4-bit ripple carry adder realized using full adders and figure 2 shows the circuit of a full adder (FA). The propagation delay of the XOR, AND and OR gates in figure 2 are 20ns, 15ns and 10ns, respectively. Assume all the inputs to the 4-bit adder are initially reset to 0.

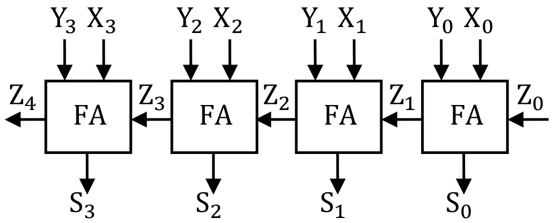


Figure I

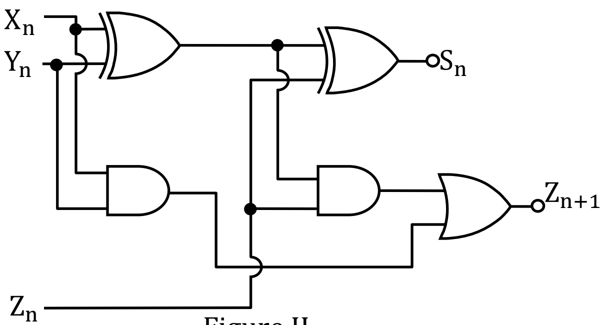
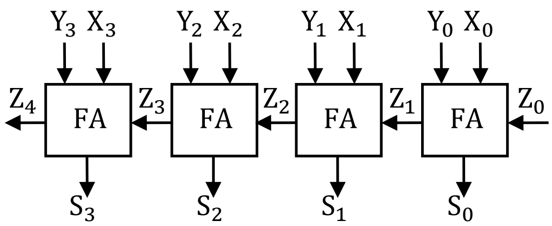


Figure II

At t = 0, the inputs to the 4-bit adder are changed to $X_3X_2X_1X_0 = 1100$, $Y_3Y_2Y_1Y_0 = 0100$ and $Z_0 = 1$. The output of the ripple carry adder will be stable at t (in ns) = ________

[Ans. *] Range: 70.0 to 70.0



Given inputs

	1	1	0	0
	0	1	0	0
Z_4	Z_3	Z_2	Z_1	1
(1)	(1)	(0)	(0)	
	0	0	0	1

Since carry = 0 for FA_1 and FA_2

$t = t_{z_3} + t_{AND} + t_{OR}$

t_{z_3} = time taken to produce carry $20 + 10 + 15 = 45$ ns

$t = 45 + 15 + 10 = 70$ ns

$FA_1 \Rightarrow t_{S_0} = 40$ ns, $t_{z_1} = 45$ ns

$FA_2 \Rightarrow$ Since carry $z_1 = 0$

$\Rightarrow$ no need to wait for carry to come, so it is executed in parallel with FA_1

$\Rightarrow t_{s_1} = 40$ ns, $t_{z_2} = 45$ ns

$FA_3 \Rightarrow$ Since carry $z_2 = 0 \Rightarrow$ Same process

$\Rightarrow t_{s_2} = 40$ ns, $t_{z_3} = 45$ ns

$FA_4 \Rightarrow$ Since carry $z_3 = 1 \Rightarrow$ it has to wait for carry to come

So $S_3 = 45 + 20 = 65$ ns
$Z_4 = 45 + 15 + 10 = 70$ ns

27. Passengers try repeatedly to get a seat reservation in any train running between two stations until they are successful. If there is 40% chance of getting reservation in any attempt by a passenger, then the average number of attempts that passengers need to make to get a seat reserved is

[Ans. *] Range: 2.4 to 2.6

Let X = Number of attempts required to get seat reserved

X	1	2	3	4	...
P(x)	$\frac{2}{5}$	$\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)$	$\left(\frac{3}{2}\right)^2\left(\frac{2}{5}\right)$	$\left(\frac{3}{5}\right)^3\left(\frac{2}{5}\right)$	...

$$\therefore E(x) = 1 \times \frac{2}{5} + 2\left(\frac{3}{5} \times \frac{2}{5}\right) + 3\left[\left(\frac{3}{2}\right)^2 \times \left(\frac{2}{5}\right)\right] + \ldots\ldots$$

$$= \frac{2}{5}\left\{1 + 2\left(\frac{3}{5}\right) + 3\left(\frac{3}{5}\right)^2 + \ldots\ldots\right\}$$

$$= \frac{2}{5}\left\{1 - \frac{3}{5}\right\}^{-2}$$

$$= \frac{2}{5}\left(\frac{2}{5}\right)^{-2} = \frac{2}{5} \times \left(\frac{5}{2}\right)^2$$

$$= 2.5$$

28. Two n-channel MOSFETs, T1 and T2, are identical in all respects except that the width of T2 is double of T1. Both the transistors are biased in the saturation region of operation, but the gate overdrive voltage (V_{GS}-V_{TH}) of T2 is double that of T1, where V_{GS} and V_{TH} are the gate-to source voltage and threshold voltage of the transistors, respectively. If the drain current and transconducatance of T1 are I_{D1} and g_{m1} respectively; the corresponding values of these two parameters for T2 are

(A) $8I_{D1}$ and $2g_{m1}$
(B) $8I_{D1}$ and $4g_{m1}$
(C) $4I_{D1}$ and $4g_{m1}$
(D) $4I_{D1}$ and $2g_{m1}$

[Ans. B]

$W_2 = 2W_1$

$V_{GS2} - V_{TH} = 2(V_{GS1} - V_{TH})$

$I_D \propto W(V_{GS} - V_{TH})^2$

$$\frac{I_{D2}}{I_{D1}} = 2 \times 2^2 = 8$$

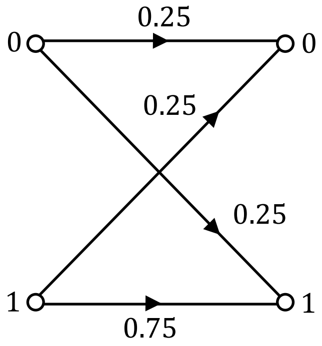
$I_{D2} = 8I_{D1}$

$g_m \propto W(V_{GS} - V_{TH})$

$$\frac{g_{m2}}{g_{m1}} = 2 \times 2 = 4$$

$g_{m2} = 4g_{m1}$

29. Consider a binary memoryless channel characterized by the transition probability diagram shown in figure.



The channel is

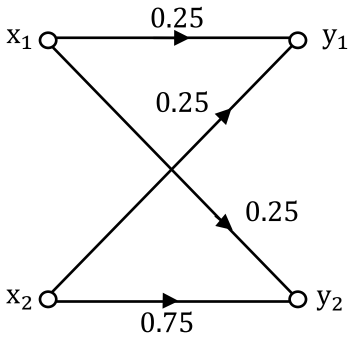
(A) lossless

(B) noiseless

(C) useless

(D) deterministic

[Ans. C]



Channel matrix:

$$P\left(\frac{Y}{X}\right) = \begin{matrix} & y_1 & y_2 \\ x_1 & 0.25 & 0.75 \\ x_2 & 0.25 & 0.75 \end{matrix} = \begin{bmatrix} 1/4 & 3/4 \\ 1/4 & 3/4 \end{bmatrix}$$

Assume $P(x_1) = P(x_2) = \frac{1}{2}$

$$P(X, Y) = \begin{bmatrix} 1/8 & 3/8 \\ 1/8 & 3/8 \end{bmatrix}$$

$P(y_1) = \frac{1}{4}, P(y_2) = \frac{3}{4}$

$$P\left(\frac{X}{Y}\right) = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$

(A) Losseless: [If $H(X/Y) = 0$]

$$H\left(\frac{X}{Y}\right) = \frac{1}{8}\log 2 + \frac{3}{8}\log 2 + \frac{1}{8}\log 2 + \frac{3}{8}\log 2$$

$= 1 \neq 0$

$\therefore$ Not Lossless

(D) Deterministic: [If $H(Y/X) = 0$]

$$H\left(\frac{Y}{X}\right) = \frac{1}{8}\log(4) + \frac{3}{8}\log\left(\frac{4}{3}\right) + \frac{1}{8}\log 4 + \frac{3}{8}\log\left(\frac{4}{3}\right)$$

$\neq 0$

$$H\left(\frac{Y}{X}\right) \neq 0$$

$\therefore$ Not Deterministic

(B) Noiseless: $\left[\text{If } H(X/Y) = H\left(\frac{Y}{X}\right) = 0\right]$

$H\left(\frac{X}{Y}\right) \neq H\left(\frac{Y}{X}\right) \neq 0$

$\therefore$ Not Noiseless

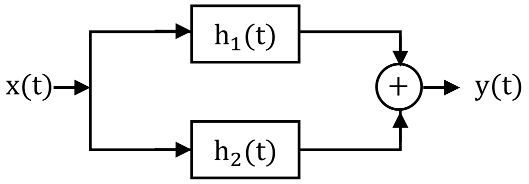
(C) Useless: [If $I(X, Y) = 0$]

$I(X, Y) = H(X) - H\left(\frac{X}{Y}\right)$

$= \frac{1}{2}\log 2 + \frac{1}{2}\log 2 - 1 = 0$

$\therefore$ Useless(Zero capacity)

30. Consider the parallel combination of two LTI systems shown in figure.



The impulse response of the systems are

$h_1(t) = 2\delta(t + 2) - 3\delta(t + 1)$

$h_2(t) = \delta(t-2)$

If the input x(t) is a unit step signal, then the energy of y(t) is

[Ans. *] Range: 7.0 to 7.0

$h(t) = h_1(t) + h_2(t)$

$x(t) = u(t)$

$y(t) = x(t) * h(t)$

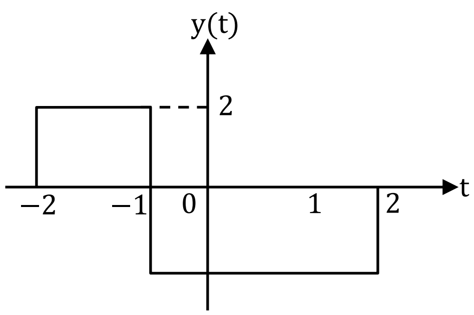
$= u(t) * [2\delta(t + 2) - 3\delta(t + 1) + \delta(t - 2)]$

Taking Laplace Transform

$Y(s)\frac{1}{s}[2e^{2s} - 3e^{s} + e^{-2s}]$

Taking Inverse laplace transform

$y(t) = 2u(t + 2) - 3u(t + 1) + u(t - 2)$



$E_{y_t} = \int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-2}^{-1} 4dt + \int_{-1}^{2} 1dt$

$= (4 \times 1) + (\times 3) = 7W$

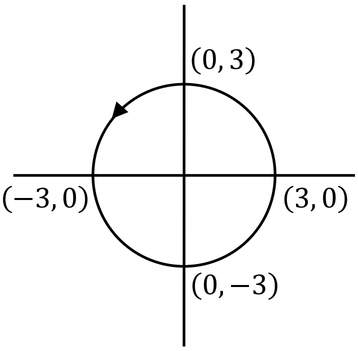
31. An integral I over a counter clock wise circle C is given by

$$I = \oint_C \frac{z^2-1}{z^2+1} e^z \, dz$$

If C is defined as |z| =3, then the value of I is

(A) $-\pi i \sin(1)$ (C) $-3\pi i \sin(1)$
(B) $-2\pi i \sin(1)$ (D) $-4\pi i \sin(1)$

[Ans. D]



Let $f(z) = \frac{z^2-1}{z^2+1} e^z = \frac{(z^2-1)e^z}{(z+i)(z-i)}$

$z = i, -i$ are simple poles lying is side 'C'

$$\underset{z=i}{\text{Res}}\, f(z) = \frac{(i^2-1)e^i}{2i} = \frac{2}{2i} e^i = ie^i$$

$$\underset{z=i}{\text{Res}}\, f(z) = \frac{((-i)^2-1)e^i}{(-2i)} = \frac{1}{i} e^{-i} = ie^{-i}$$

∴ By Cauchy residue theorem,

$$\oint_c \frac{z^2-1}{z^2+1} e^z dz = 2\pi i\left(ie^i - ie^{-i}\right)$$
$$= -2\pi\left(e^i - e^{-i}\right)$$
$$= -2\pi\{[\cos(1) + i\sin(1)] - [\cos(1) - i\sin(1)]\}$$
$$= -4\pi i \sin(1)$$

32. An electron (q_1) is moving in free space with velocity 10^5 m/s towards a stationary electron (q_2) far away. The closest distance that this moving electron gets to the stationary electron before the repulsive force diverts its path is ____ $\times 10^{-8}$m

[Given, mass of electron $m = 9.11 \times 10^{-31}$kg, charge of electron $e = -1.6 \times 10^{-19}$C, and permittivity $\varepsilon_0 = (1/36\pi) \times 10^{-9}$ F/m].

[Ans. *] Range: 4.55 to 5.55

As electron (q_1) moving with velocity 10^5 m/s i.e it is having kinetic energy

$$\text{K.E} = \frac{1}{2} mV^2$$

As electron q_2 which is at rest having potential energy P.E = qV.

The moving electron continue it's motion with out deflection until P .E of q_2 = K . E of q_1.

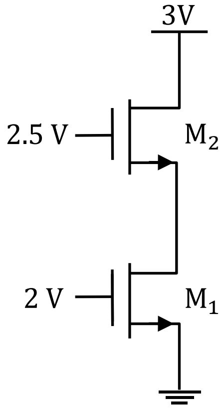
$$\frac{1}{2} mV^2 = qV \text{ (V is voltage which is same as of potential of charge at rest)}$$

$\frac{1}{2}mV^2 = q\cdot\frac{q}{4\pi\varepsilon_0 R}$ (R is the shortest distance)

$R = \frac{q^2\times 2}{4\pi\varepsilon_0\times mV^2} = \frac{(1.6\times10^{-19})^2\times 2}{4\pi\times\frac{10^{-9}}{36\pi}\times 9.1\times10^{-31}\times(10^5)^2} = 5.06\times10^{-8}\text{m}$

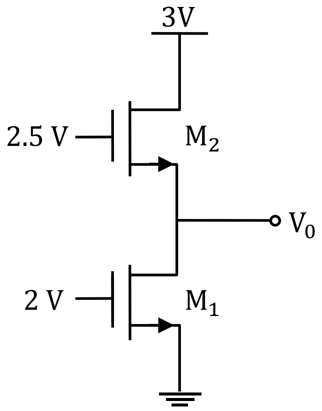
$R = 5.06\times10^{-8}\text{m}$

33. Assuming that transistors M_1 and M_2 are identical and have a threshold voltage of 1V, the state of transistors M_1 and M_2 are respectively



(A) Saturation, Saturation
(B) Linear, Linear
(C) Linear, Saturation
(D) Saturation, Linear

[Ans. C]



Given $V_{th} = 1\text{V}$

Here both the transistors are 'ON'

For M_2:

$V_G - V_{th} < V_D [1.5 < 3]$

$\Rightarrow M_2$ is in saturation

For M_1:

Let us assume M_1 is in saturation

$(I_D)M_2 = (I_D)M_1$

$(2.5 - V_0 - 1)^2 = (2-1)^2 [\because I_D \propto (V_{GS} - V_{th})^2]$

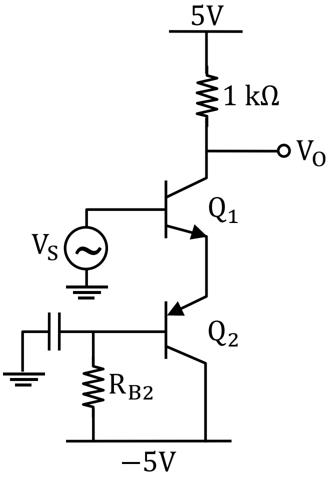
$\therefore V_0 = 0.5$

$V_{GS} - V_{th} > V_{DS} [1 > 0.5]$

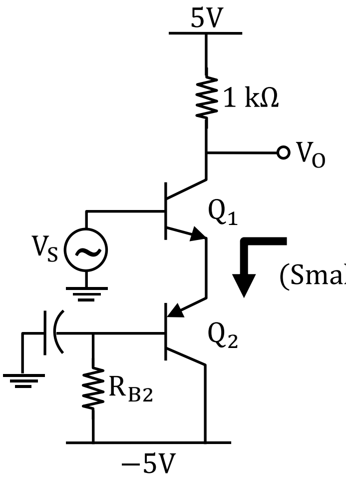
$\Rightarrow$ our assumption is wrong

$\therefore M_1$ is in triode region
$M_2 \rightarrow$ saturation
$M_1 \rightarrow$ triode

34. In the circuit shown, transistor Q_1 and Q_2 are biased at a collector current of 2.6 mA. Assuming the transistor current gains are sufficiently large to assume collector current equal to emitter current and thermal voltage of 26 mV, the magnitude of voltage gain V_o/V_s in the mid band frequency range is _______ (up to second decimal place).



[Ans. *] Range: 49.0 to 51.0



$$A_V = \frac{V_0}{V_s} = \frac{-g_{m1}R_C}{1 + R_{m1}R_{eq}}$$

Herer $g_{m1} = g_{m2} = \frac{1}{R_{eq}} = \frac{I_{CQ}}{V_T} = \frac{2.6mA}{2.6mV} = 10^{-1}$

$$\therefore A_V = \frac{-g_{m1}RC}{2} = \frac{-0.1 \times 1000}{2} = -50$$

or

Method 2:

$V_S = 2\ V_{be}$

$V_0 = -i_C R_C$

$$\frac{V_0}{V_S} = \frac{-i_C R_C}{2V_{be}}$$
$$= \frac{-g_m}{2} RC$$
$$= -50$$

35. A second order LTI system is described by the following state equation.

$$\frac{d}{dt}x_1(t) - x_2(t) = 0$$
$$\frac{d}{dt}x_2(t) + 2x_1(t) + 3x_2(t) = r(t)$$

When $x_1(t)$ and $x_2(t)$ are the two state variables and $r(t)$ denotes the input. The output $c(t) = x_1(t)$. The system is

(A) undamped (oscillatory)
(B) under damped
(C) critically damped
(D) over damped

[Ans. D]

$\dot{x}_1 - x_2 = 0 \Rightarrow \dot{x}_1 - x_2 \ldots\ldots\ldots$ ①

$\dot{x}_2 + 2x_1 + 3x_2 = r$

$\dot{x}_2 = r - 2x_1 - 3x_2 \ldots\ldots\ldots$ ②

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$C = x_1$

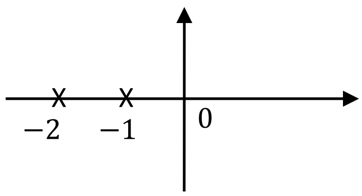
$$[C] = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$TF = C \frac{\text{Adj } [SI - A]}{|SI - A|} B + D$$

$$[SI - A] = \begin{bmatrix} S & -1 \\ 2 & S+3 \end{bmatrix} \text{Adj } [SI - A] = \begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix}$$

$$TF = \frac{[1 \quad 0] \begin{bmatrix} S+3 & +1 \\ -2 & S \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}}{S(S+3) + 2} = \frac{[1 \quad 0] \begin{bmatrix} 1 \\ S \end{bmatrix}}{S^2 + 3S + 2} = \frac{1}{S^2 + 3S + 2}$$

CE $S^2 + 3S + 2 = 0$



Over damped system

36. The un-modulated carrier in an AM transmitter is 5kW. This carrier is modulated by a sinusoidal modulating signal. The maximum percentage of modulation is 50%. If it is reduced to 40%, then the maximum unmodulated carrier power (in kW) that can be used without over loading the transmitter is _____

[Ans. *] Range: 5.19 to 5.23

$P_C' = 5$ kW

$\mu = 0.5$

$P_t = 5000\left[1 + \frac{0.25}{2}\right]$

$= 5000 \times 1.125$

$P_t = 5625\text{ W}$

$\mu = 0.4$

$5625 = P_C'\left[1 + \frac{0.16}{2}\right]$

$5625 = P_C'[1.08]$

$P_C' = \frac{5625}{1.08} = 5208\text{ W}$

37. For a particular intensity of incident light on a silicon pn junction solar cell, the photocurrent density (J_L) is 2.5 mA/cm^2 and the open-circuit voltage (V_{OC}) is 0.451V. Consider thermal voltage (V_T) to be 25 mV. If the intensity of the incident light is increased by 20 times, assuming that the temperature remains unchanged, V_{OC} (in volts) will be ________

[Ans. *] Range: 0.51 to 0.54

The open circuit voltage of a solar cell is given by

$V_{oc} = \frac{kT}{q}\ln\left[\frac{I_{ph}}{I_{so}}\right]$;

I_{ph} = Photo current

I_{ph} J_L So

I_{ph} = 2.5 K (K is constant)

I_{s0} = Reverse saturation current

$\Rightarrow V_{oc} = V_T \ln\left[\frac{I_{pg}}{I_{so}}\right]$

For a given area of operation, the above equation can be rewritten as

$V_{oc} = V_T \ln\left[\frac{I_{ph}}{I_{so}}\right]$

$\Rightarrow V_{oc} = V_T \ln\left[\frac{I_L}{I_s}\right]$

$\therefore 0.451 = 25 \times 10^{-3} \ln\left[\frac{2.5 \times 10^{-3}K}{I_S}\right] \ldots\ldots (1)$

Now intensity of light is increased by 20 times

$\therefore V_{OC}' = V_T \ln\left[\frac{20 \times 2.5 \times 10^{-3}K}{I_S}\right] \ldots\ldots (2)$

By (1) − (2) we get

$0.451 - V_{oc}' = V_T \ln\left[\frac{2.5 \times 10^{-3}K}{I_S}\right] - V_T \ln\left[\frac{50 \times 10^{-3}L}{I_S}\right]$

$\Rightarrow 0.451 - V_{oc}' = V_T \ln\left[\frac{2.5 \times 10^{-3}K}{I_S} \times \frac{I_S}{50 \times 10^{-3}K}\right]$

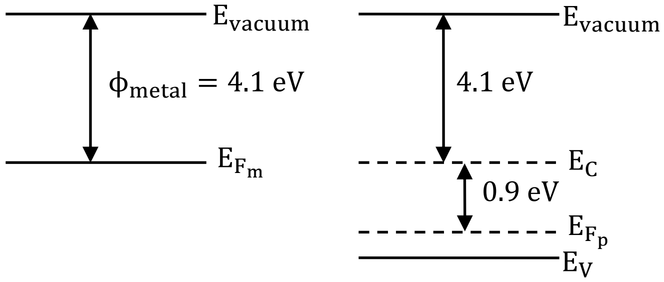
$\Rightarrow 0.451 - V_{oc}' = 25 \times 10^{-3} \ln(0.05)$

$\Rightarrow 0.451 - V_{oc}' = -0.07489$

$\Rightarrow V_{oc} = 0.526\text{ V}$

38. A MOS capacitor is fabricated on p-type Si (silicon) where the metal work function is 4.1eV and electron affinity of Si is 4.0eV, $E_C - E_F = 0.9eV$; where E_C and E_F are conduction band minimum and the Fermi energy levels of Si, respectively. Oxide $\varepsilon_r = 3.9$, $\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm, oxide thickness $t_{ox} = 0.1$ μm and electron charge $q = 1.6 \times 10^{-19}$C. If the measured flat band voltage of this capacitor is −1V, then the magnitude of the fixed charge at the oxide semiconductor interface, in nC/cm^2, is _______

[Ans. *] Range: 6.85 to 6.95



So $\phi_{semiconductor} = 4 + 0.9 = 4.9$ eV

{Work function $\Rightarrow E_{Vacuum} - E_{Fermilevel}$}

$V_{FB} = \phi_{ms} - \frac{q_{ox}}{C_{ox}}$

$-1 = -0.8 - \frac{q_{ox}}{C_{ox}}$

$\begin{cases} V_{FB} \rightarrow \text{Flat band voltage} \\ \phi_{ms} \rightarrow \phi_m - \phi_s \end{cases}$

{$\frac{q_{ox}}{C_{ox}} \Rightarrow$ potential developed due to charge at surface

$0.2 = \frac{q_{ox}}{C_{ox}}$

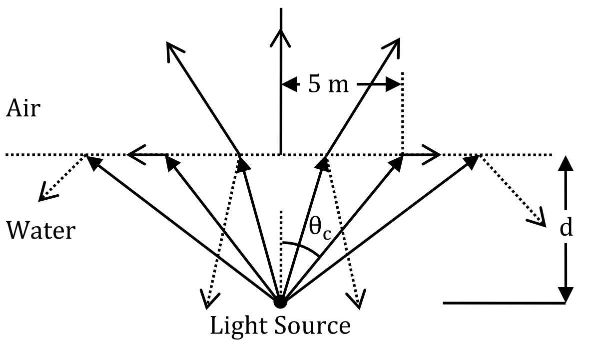
$q_{ox} = 0.2\ C_{ox}$

$= 0.2 \frac{\varepsilon_{ox}}{t_{ox}}$

$= 0.2 \times \frac{3.9 \times 8.85 \times 10^{-14}}{10^{-5}}$

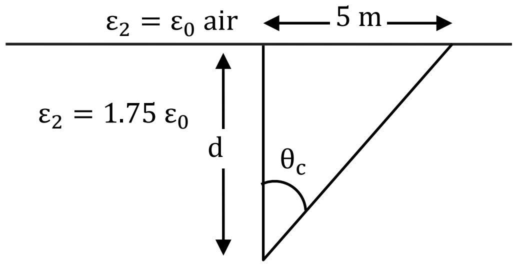
$q_{ox} = 6.903$ nC/cm^2

39. The permittivity of water at optical frequencies is $1.75\varepsilon_0$. It is found that an isotropic light source at a distance d under water forms an illuminated circular area of radius 5 m, as shown in the figure. The critical angle is θ_c



The value of d (in meter) is _______

[Ans. *] Range: 4.2 to 4.4



$\sin\theta_c = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}$; $\sin\theta_c = \frac{1}{\sqrt{1.75}}$

$\therefore \theta_c = 49°$

Form the triangle $\tan\theta_c = \frac{5}{d}$

$d = \frac{5}{\tan(49)} = \frac{5}{1.15} = 4.34$ m

40. The minimum value of the function $(x) = \frac{1}{3}x(x^2 - 3)$ in the interval $-100 \le x \le 100$ occurs at $x =$ ________

[Ans. *] Range: -100.01 to -99.99

$f(x) = \frac{1}{3}x(x^2 - 3) = \frac{x^3}{3} - x \text{in} [-100, 100]$

$f'(x) = x^2 - 1 = 0$

$\Rightarrow x = 1, -1$ are critical points

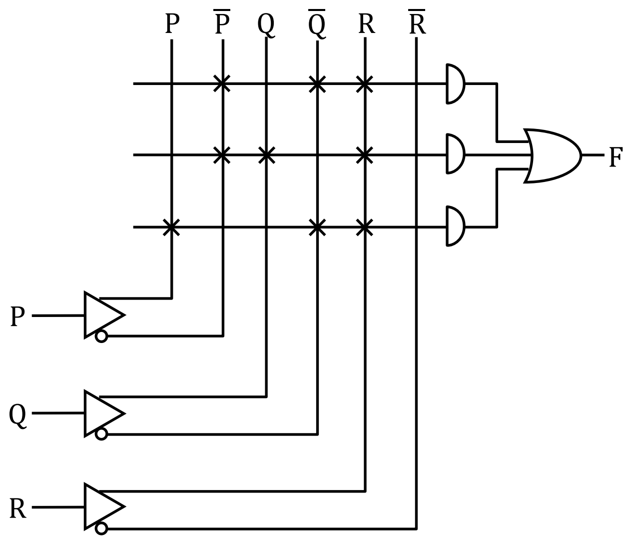
$f(1) = -\frac{2}{3}, f(-1) = \frac{2}{3}$

$f(-100) = -333233.33$

$f(100) = 333233.33$

$\therefore$ The minimum values occurs at $x = -100$

41. A programmable logic array (PLA) is shown in the figure.



The Boolean function F implemented is

(A) $\overline{P}\,\overline{Q}R + \overline{P}QR + P\overline{Q}\,\overline{R}$

(B) $(\overline{P} + \overline{Q} + R)(\overline{P} + Q + R) + (P + \overline{Q} + \overline{R})$

(C) $\overline{P}\,\overline{Q}R + \overline{P}QR + P\overline{Q}\,\overline{R}$

(D) $(\overline{P} + \overline{Q} + R)(\overline{P} + Q + R) + (P + \overline{Q} + R)$

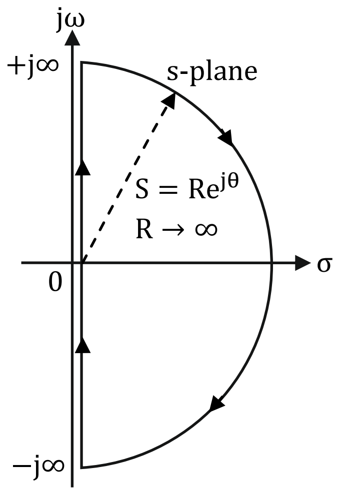
[Ans. C]

$F = \overline{P}\,\overline{Q}R + \overline{P}QR + P\overline{Q}\,\overline{R}$

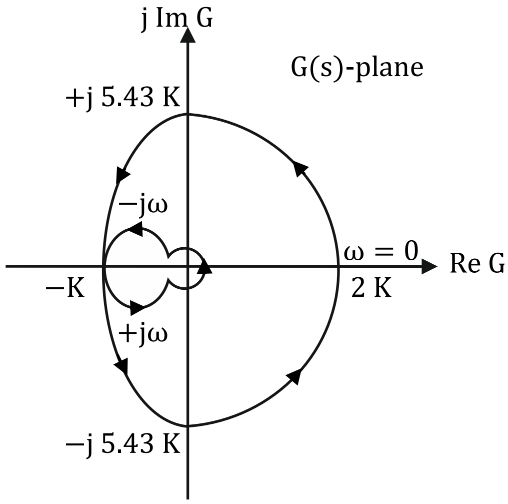
42. A unity feedback control system is characterized by the open loop transfer function

$$G(s) = \frac{10K(s+2)}{s^3 + 3s^2 + 10}$$

The Nyquist path and the corresponding Nyquist plot of G(s) are shown in the figures below.



Nyquist Path for G(s)



Nyquist plot of G(s)

If $0 < K < 1$, then number of poles of the closed loop transfer function that lie in the right half of the s-plane is

(A) 0
(B) 1
(C) 2
(D) 3

[Ans. C]

$$G(s) = \frac{10k\,(s+2)}{s^3+3s^2+10}$$

$$G(s) = \frac{10k(s+2)}{(s+3.72)(s-0.36+j1.5)(s-0.36-j1.5)}$$

$P = 2$(Two poles in the RHS)

if $K < 1$, The number of encircleements about $(-1, j0)$ is 0

$N = P - Z$

$N = 2 - 0 = 2$

$\Rightarrow$ 2 CL poles lies in the RHS-plane.

43. A unity feedback control system is characterized by the open loop transfer function

$$G(s) = \frac{2(s+1)}{s^3+ks^2+2s+1}$$

The value of k for which the system oscillates at 2 rad/s is_______

[Ans. *] Range: 0.74 to 0.76

$$G(s) = \frac{2(s+1)}{s^3+ks^2+2s+1}, \text{given } \omega = 2\text{ rad/sec}$$

$CE \Rightarrow S^3 + kS^2 + 4S + 3 = 0$

S^3	1	4
S^2	K	3
S^1	$\frac{4k-3}{k}$	
S^0	3	

For marginal stable $\frac{4k-3}{k} = 0 \Rightarrow K = \frac{3}{4} = 0.75$

44. The values of the integrals

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dy\right)dx$$

and

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dx\right)dy$$

are

(A) same and equal to 0.5

(B) same and equal to -0.5

(C) 0.5 and -0.5, respectively

(D) -0.5 and -0.5, respectively

[Ans. C]

The values of the integral

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dy\right)dx$$

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dx\right)dy$$

$$\int_0^1\left(\int_0^1 \frac{x-y}{(x+y)^3}dy\right)dx = \int_0^1\left\{\frac{2x-(x+y)}{(x+y)^3}dy\right]dx$$

$$= \int_0^1\left\{\left[\frac{2x}{(x+y)^3}-\frac{1}{(x+y)^2}\right]dy\right\}dx$$

$$= \int_0^1\{[2x\,(x+y)^{-3}-(x+y)^{-2}]dy\}\,dx$$

$$= \int_0^1\left\{\frac{2x(x+y)^{-2}}{-2}-\frac{(x+y)^{-1}}{-1}\right\}_0^1 dx$$

$$= \int_0^1\left\{\frac{-x}{(x+y)^2}+\frac{1}{x+y}\right\}_0^1 dx$$

$$= \int_0^1\left\{\left[\frac{-x}{(x+1)^2}+\frac{1}{x+1}\right]-\left[\frac{-1}{1}+\frac{1}{x}\right]\right\}dx$$

$$= \int_0^1\left\{\frac{-x+x+1}{(x+1)^2}\right\}dx$$

$$= \int_0^1 \frac{1}{(x+1)^2}dx$$

$$= \left(\frac{-1}{x+1}\right)_0^1$$

$$= \left(\frac{-1}{2}\right)-(-1)$$

$$= \frac{1}{2}$$

$$= \int_0^1\left[\int_0^1 \frac{x-y}{(x+y)^3}dx\right]dy = \int_0^1\left[\frac{(x+y)-2y}{(x+y)^3}dx\right]dy$$

$$= \int_0^1\left\{\left[\frac{1}{(x+y)^2}-\frac{2y}{(x+y)^3}\right]dx\right\}dy$$

$$= \int_0^1\left\{-\frac{1}{x+y}+\frac{y}{(x+y)^2}\right\}_0^1 dy$$

$$= \int_0^1\left\{\left[-\frac{1}{y+1}+\frac{y}{(y+1)^2}\right]-\left[-\frac{1}{y}+\frac{1}{y}\right]\right\}dy$$

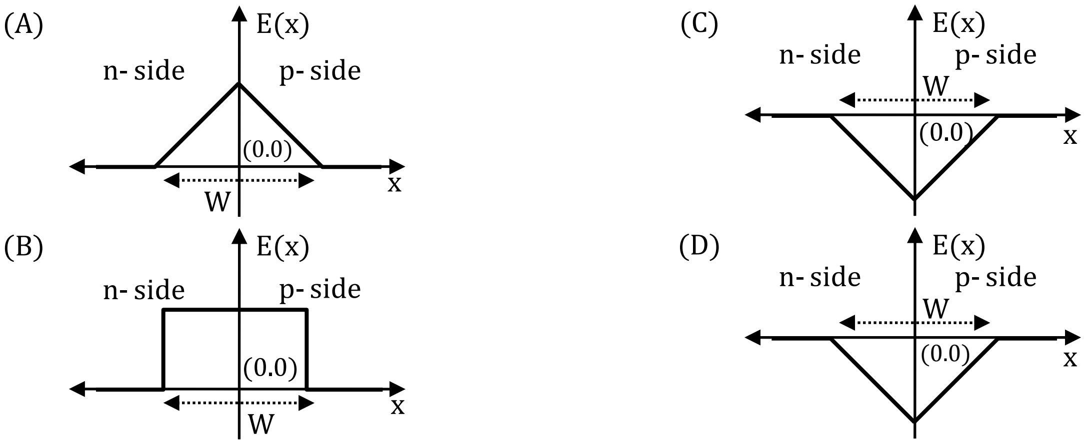
$$= \int_0^1\left\{\left[-\frac{1}{y+1}+\frac{y}{(y+1)^2}\right]\right\}dy$$

$$= \int_0^1\left[\frac{-(y+1)+y}{(y+1)^2}\right]dy$$

$$= \int_0^1 \frac{-dy}{(1+y)^2}$$

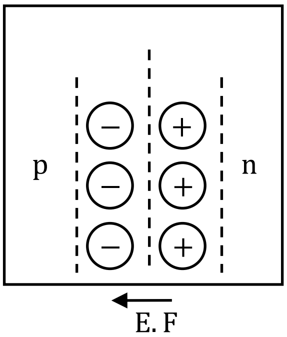
$$= \left(\frac{1}{1+y}\right)_0^1$$
$$= \frac{1}{2} - 1 = -\frac{1}{2}$$

45. An abrupt pn junction (located at x=0) is uniformly doped on both p and n sides. The width of the depletion region is W and the electric field variation in the x-direction is E(x). Which of the following figures represents the electric field profile near the pn junction.



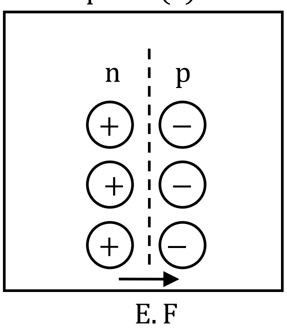
[Ans. A]

We know Electric field is maximum at middle of junction so options (B), (D) can be eliminated. In any p-n junction, Electric field is always from n to p (since $V_n > V_p$)



We know $E = -\nabla V$

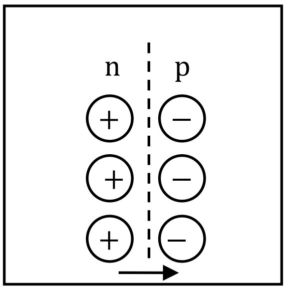
Take option (a):



$$V_p - V_N = -\int E.dx = -ve \text{ (in option (A), } E_0 \text{ is positive)}$$

$\Rightarrow V_N > V_P$ (which is always true)

Take option (C):



$V_P - V_N = -\int E.dx$ {In option (c), E_0 is negative}

$= +Ve$

$\Rightarrow V_p > V_N$ (Not possible)

46. A modulating signal given by $x(t) = 5\sin(4\pi 10^3 t - 10\pi \cos 2\pi 10^3 t)$ V is fed to a phase modulator with phase deviation constant $k_p = 5$ rad/V. If the carrier frequency is 20 kHz, the instantaneous frequency (in kHz) at t = 0.5ms is ________

[Ans. *] Range: 69.9 to 70.1

$s(t) = A_C \cos[2\pi f_c t + k_p m(t)]$

$$f_i = f_c + \frac{k_p}{2\pi}\frac{d}{dt}x(t)$$

$$= 20\text{ k} + \frac{5}{2\pi} \times 5\frac{d}{dt}(\sin 4\pi 10^3 t - 10\pi \cos 2\pi 10^3 t)$$

$$= 20\text{ k} + \frac{25}{2\pi} \times [\cos(4\pi 10^3 t - 10\pi\cos 2\pi 10^3 t)\,(4\pi 10^3 + 10\pi \sin 2\pi 10^3 t \times 2\pi 10^3)]$$

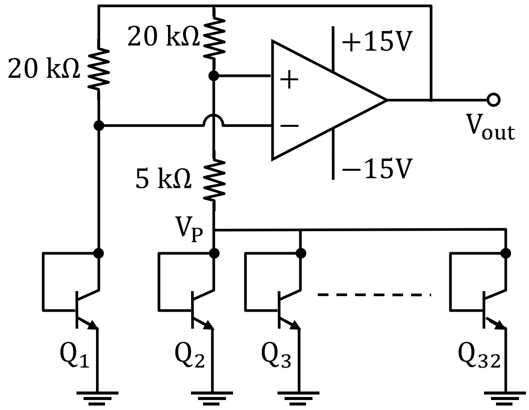
$$f_{i(t=0.5\text{ ms})} = 20\text{k} + \frac{25}{2\pi} \times \cos(4\pi + 10\pi) \times 4\pi \times 10^3$$

$$= 20\text{ k} + \frac{25}{2\pi} \times 4\pi \times 10^3$$

$$= 20\text{ k} + 50\text{ k}$$

$$= 70\text{ k}$$

47. In the voltage reference circuit shown in the figure, the op-amp is ideal and the transistors Q_1, Q_2 ..., Q_{32} are identical in all respects and have infinitely large values of common – emitter current gain (β). The collector current (I_c) of the transistors is related to their base emitter voltage (V_{BE}) by the relation $I_C = I_S \exp(V_{BE}/V_T)$; where I_s is the saturation current. Assume that the voltage V_P shown in the figure is 0.7V and the thermal voltage $V_T = 26$ mV



The output voltage V_{out} (in volts) is ______

[Ans. *] Range: 1.1 to 1.2

From Fig. $V_+ = V_{be1}$ (Q_1 transistor)

$V_+ = V_-$ (Virtial short)

$$\frac{V_{out} - V_{be1}}{20k} = I_S e^{\frac{V_{be1}}{26m}} \ldots\ldots\ldots ①$$

$$\frac{V_{out} - V_{be1}}{20k} = \frac{V_{be1} - 0.7}{5k} = 31 I_S e^{\frac{V_{be1}}{26m}} \ldots\ldots\ldots ②$$

$$\frac{②}{①} \Rightarrow 1 = 31 e^{\frac{0.7 - V_{bel}}{26\,m}}$$

$\Rightarrow V_{be1} = 0.789283667$

Substituting V_{be1} in ②

$$\frac{V_{out} - 0.78928}{20k} = \frac{0.78928 - 0.7}{5k}$$

$\Rightarrow V_{out} = 1.14642V$

48. Consider an LTI system with magnitude response

$$|H(f)| = \begin{cases} 1 - \frac{|f|}{20}, & |f| \le 20 \\ 0. & |f| > 20 \end{cases}$$

and phase response

$\text{Arg}[H(f)] = -2f.$

If the input to the system is

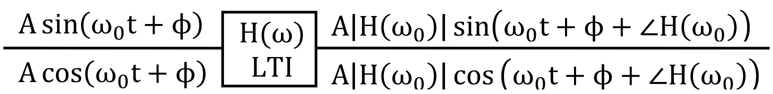
$$x(t) = 8\cos\left(20\pi t + \frac{\pi}{4}\right) + 16\sin\left(40\pi t + \frac{\pi}{8}\right) + 24\cos\left(80\pi t + \frac{\pi}{16}\right)$$

Then the average power of the output signal y(t) is ________

[Ans. *] Range: 7.95 to 8.05

$$|H(f)| = \begin{cases} 1 - \frac{|f|}{20}; & |f| \le 20 \\ 0; & |f| > 20 \end{cases}$$

$\angle H(f) = -2f$



For the given first input signal $8\cos\left(20\pi t + \frac{\pi}{4}\right)$

$f_0 = 10\text{ Hz}$

$|H(f_0)| = 1 - \frac{1}{2} = \frac{1}{2}$

$\angle H(f_0) = -2 \times 10 = -20$

The output is $= \left(8 \times \frac{1}{2}\right) \cos\left(20\pi t + \frac{\pi}{4} - 20\right)$

$= 4\cos\left(20\pi t + \frac{\pi}{4} - 20\right)$

For the given second input signal $16\sin\left(40\pi t + \frac{\pi}{8}\right)$

$f_0 = 20\text{ Hz}$

$|H(f_0)| = 0$

$\angle H(f_0) = -40°$

The output is zero

For the given third input signal $24\cos\left(80\pi t + \frac{\pi}{16}\right)$

$f_0 = 40\text{ Hz}$

$H(f_0) = 0$ as $f_0 > 20$

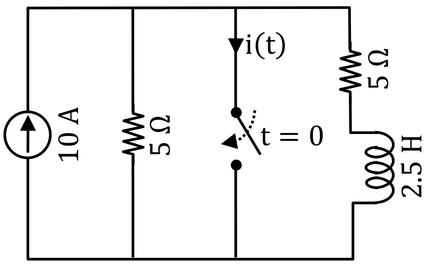
$\angle H(f_0) = -80°$

The output is zero

$y(t) = 4\cos\left(20\pi t + \frac{\pi}{4} - 20°\right) + 0 + 0 = 4\cos\left(20\pi t + \frac{\pi}{4} - 20°\right)$

$P_y(t) = \frac{4^2}{2} = 8\text{ W}$

49. The switch in the circuit, shown in the figure, was open for a long time and is closed at t =0

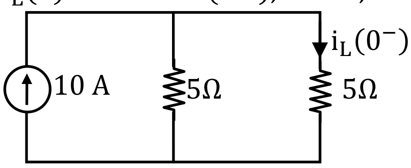


The current i(t) (in ampere) at t = 0.5 seconds is

[Ans. *] Range: 8.0 to 8.3

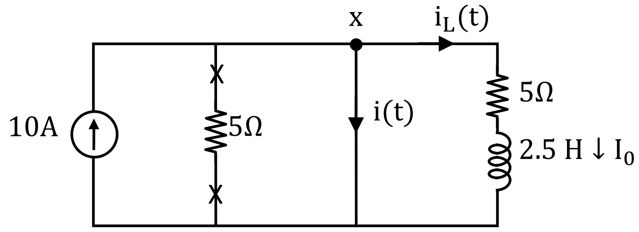
For $t < 0$, switch is opened (steady state)

$i_L(0) \Rightarrow t = 0 - (S.S), R \to R, L \to S.C$



$i_L(0^-) = \frac{10}{2} = 5A = i_L(0^+) = I_0$

For $t \geq 0$, switch is closed



For R-L Source free circuit $i_L(t) = i_0 e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{2.5}{5} = \frac{1}{2}\text{sec}$

$i_L(t) = 5\,e^{-2t}\text{Amps}$

By KCL at 'x'

$10 = i_L(t) + i(t)$

$i(t) = 10 - iL(t)$

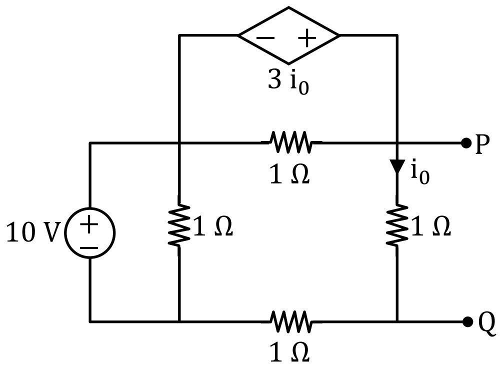
$\quad = 10 - 5\,e^{-2t}\text{ Amp}$

At $t = 0.5$ sec

$i(t) = 10 - 5\,e^{-1}$

$\quad = 8.16\text{Amp}$

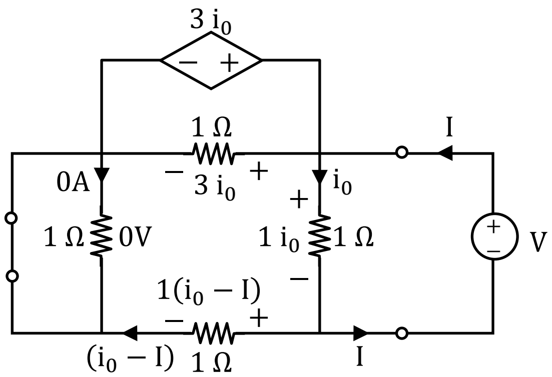
50. Consider the circuit shown in figure



The Thevenin equivalent resistance (in Ω) across P-Q is ________

[Ans. *] Range: -1.01 to -0.99

Evaluation of R_{th} by case (3) approach



→ here, $V = 1\,i_0$ ……… ①

By KVL ⇒ $0 + 3i_0 - 1.i_0 - 1(i_0 - I) = 0$

⇒ $3i_0 - i_0 - i_0 + I = 0$

$\Rightarrow i_0 + I = 0$

$\Rightarrow I = -i_0$ ……… ②

From ① and ②

$R_N = R_{th} = \frac{V}{I} = \frac{+i_0}{-i_0} = -1\Omega$

51. Standard air filled rectangular waveguides of dimensions a = 2.29 cm and b = 1.02 cm are designed for radar applications. It is desired that these waveguides operate only in the dominant TE_{10} mode with the operating frequency at least 25% above the cut-off frequency of the TE_{10} mode but not higher than 95% of the next higher cutoff frequency. The range of the allowable operating frequency f is

(A) $8.19\text{ GHz} \le f \le 13.1\text{ GHz}$
(B) $8.19\text{ GHz} \le f \le 12.45\text{ GHz}$
(C) $6.55\text{ GHz} \le f \le 13.1\text{ GHz}$
(D) $1.64\text{ GHz} \le f \le 10.24\text{ GHz}$

[Ans. B]

Given: a=2.29cm, b=1.02cm

$f_c\big|TE_{10} = \frac{c}{2a} = \frac{3\times10^8}{2\times2.29\times10^{-2}} = 6.5\text{ GHz}$

Since, $b < \frac{a}{2}$ next higher order mode is TE_{20}

$f_c\big|TE_{20} = \frac{c}{2}\times\frac{2}{a} = \frac{c}{a} = \frac{3\times10^8}{2.29\times10^{-2}} = 13.1\text{ GHz}$

So, the range of allowable operating frequency is

$1.25\, f_c|_{TE_{10}} \le f \le 0.95 f_c|_{TE_{20}}$

i.e. $8.19\text{GHz} \le f \le 12.45\text{GHz}$.

52. If the vector function $\vec{F} = \hat{a}_x(3y - k_1z) + \hat{a}_y(k_2x - 2z) - \hat{a}_z(k_3y + z)$ is irrotational, then the values of the constants k_1, k_2 and k_3 respectively, are

(A) 0.3, −2.5, 0.5
(B) 0.0, 3.0, 2.0
(C) 0.3, 0.33, 0.5
(D) 4.0, 3.0, 2.0

[Ans. B]

Given

$\bar{F} = (3y - k_1z)\vec{i} + (k_2x - 2z)\vec{j} - (k_3y + z)\vec{k}$

Curl $\vec{F} = 0$

$$\Rightarrow \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 3y - k_1z & k_2x - 2z & -k_3y - z \end{vmatrix} = 0$$

$\Rightarrow (-k_3 + 2)\vec{i} - \vec{j}(0 - k_1) + \vec{k}(k_2 - 3) = 0$

$k_3 = 2; k_1 = 0; k_2 = 3$

53. The transfer function of a causal LTI system is $H(s) = 1/s$. If the input to the system is $x(t) = [\sin(t)/\pi t]u(t)$; where u(t) is a unit step function. The system output y(t) as $t \to \infty$ is ______

[Ans. *] Range: 0.45 to 0.55

$$H(s) = \frac{1}{s}$$

$$x(t) = \frac{\sin t}{\pi t} u(t)$$

$$\sin t\ u(t) \leftrightarrow \frac{1}{s^2+1}$$

$$\frac{\sin t\ u(t)}{t} \leftrightarrow \int_s^{\infty} \frac{1}{s^2+1} ds = \tan^{-1}(s)|_s^{\infty} = \frac{\pi}{2} - \tan^{-1}(s)$$

$$X(s) = \frac{1}{\pi}\left[\frac{\pi}{2} - \tan^{-1}(s)\right]$$

$$= \frac{1}{2} - \frac{1}{\pi}\tan^{-1}(s)$$

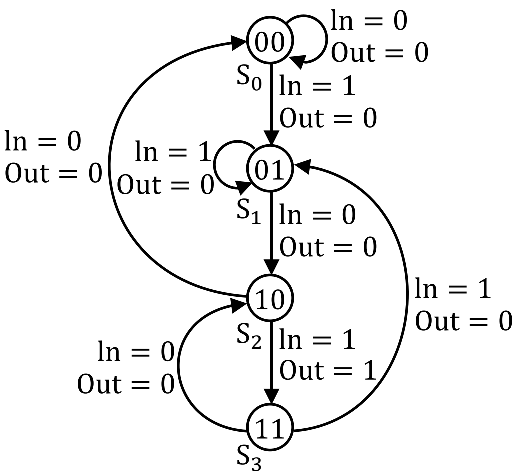
$$H(s) = \frac{Y(s)}{X(s)}$$

$$\Rightarrow Y(s) = X(s)H(s) = \left[\frac{1}{2} - \frac{1}{\pi}\tan^{-1}(s)\right]\frac{1}{s}$$

$$y(\infty) = \lim_{s-0} sY(s) = \lim_{s\to 0}\left[\frac{1}{2} - \frac{1}{\pi}\tan^{-1}(s)\right]$$

$$= \frac{1}{2}$$

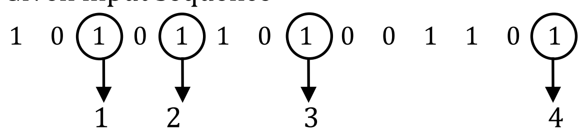
54. The state diagram of a finite state machine (FSM) designed to detect an overlapping sequence of three bits is shown in the figure. The FSM has an input 'In' and an output 'out'. The initial state of the FSM is S_0.



If the input sequence is 10101101001101, starting with the left most bit, then the number of times 'Out' will be 1 is _____

[Ans. *] Range: 4 to 4

Given input sequence



Number of times out will be 1 is **4**

55. The signal $x(t) = \sin(14000\pi t)$, where t is in seconds, is sampled at a rate of 9000 samples per second. The sampled signal is the input to an ideal lowpass filter with frequency response H(f) as follows:

$$H(f) = \begin{cases} 1, |f| \le 12\text{kHz} \\ 0, |f| > 12\text{kHz} \end{cases}$$

What is the number of sinusoids in the output and their frequencies in kHz?

(A) Number =1, frequency = 7
(B) Number = 3, frequencies = 2, 7, 11
(C) Number = 2, frequencies = 2, 7
(D) Number = 2, frequencies = 7, 11

[Ans. B]

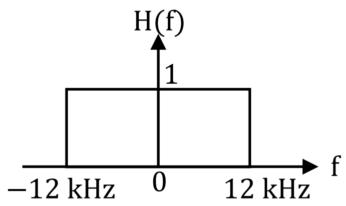
Given $x(t) = \sin(14000\pi t)$

$f_m = 7$ kHz,

$f_s = 9$ kHz,

The frequency of sampled signal are $\pm f_m \pm nf_s$

= 7 kHz, 2 kHz, 16 kHz, 11 kHz, 25 kHz, …………



The output frequencies of the filter are = 7 kHz, 2 kHz, 11 kHz