

GATE 2011

Instrumentation Engineering

Q.1 - Q.25 Carry One Mark each.

1. The matrix $M = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ has eigenvalues $-3, -3, 5$. An eigenvector corresponding to the eigenvalue 5 is $[1 \ 2 \ -1]^T$. One of the eigenvectors of the matrix M^3 is

- (A) $[1 \ 8 \ -1]^T$ (C) $[1 \ \sqrt[3]{2} \ -1]^T$
 (B) $[1 \ 2 \ -1]^T$ (D) $[1 \ 1 \ -1]^T$

[Ans. B]

$$Mx = \lambda x$$

$$\Rightarrow Mx = \lambda Ix$$

$$(M - \lambda I)x = 0 \text{ for non-zero 'x'}$$

For M^3 , Eigen values become λ^3

$$\therefore (M^3 - \lambda^3 I)x' = 0$$

$$\Rightarrow (M^3 + \lambda M + \lambda^2 I)(M - \lambda I)x' = 0$$

\therefore for $x' = x$, the above condition is satisfied.

2. The contour integral

$$\oint_C e^{1/z} dz$$

with C as the counter-clockwise unit circle in the z-plane is equal to

- (A) 0 (C) $2\pi\sqrt{-1}$
 (B) 2π (D) ∞

[Ans. C]

$$f(z) = \oint_C e^{1/z} dz$$

$$= \oint_C \left(1 + \frac{1}{z} + \frac{1}{2z^2} + \frac{1}{6z^3} + \dots \right) dz$$

The only pole of $f(z)$ is at $z = 0$, which lies within $|z| = 1$

$$\therefore \int f(z) dz = 2\pi i (\text{residue})$$

Note: Residue of $f(z)$ at $z = 0$ is coefficient of $1/z$ i.e. 1, here.

3. Consider the signal

$$x(t) = \begin{cases} e^{-t}, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

Let $X(\omega)$ denote the Fourier transform of this signal. The integral

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega \text{ is}$$

- (A) 0 (C) 1
 (B) $\frac{1}{2}$ (D) ∞

[Ans. C]

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) e^{j\omega t} d\omega$$

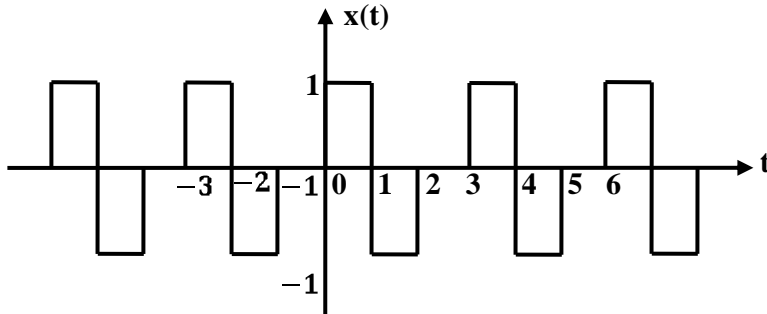
$$t = 0 \Rightarrow \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(\omega) d\omega = x(0) = 1$$

4. The continuous-time signal $x(t) = \sin \omega_0 t$ is a periodic signal. However, for its discrete-time counterpart $x[n] = \sin \omega_0 n$ to be periodic, the necessary condition is
- (A) $0 \leq \omega_0 < 2\pi$ (C) $\frac{2\pi}{\omega_0}$ to be a ratio of integers
 (B) $\frac{2\pi}{\omega_0}$ to be an integer (D) none

[Ans. C]

For $x[n]$ to be periodic, $\frac{2\pi}{\omega_0}$ should be rational.

5. Consider a periodic signal $x(t)$ as shown below



It has a Fourier series representation $x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j(2\pi/T)kt}$

Which one of the following statements is **TRUE**?

- (A) $a_k = 0$, for k odd integer and $T = 3$ (C) $a_k = 0$, for k even integer and $T = 6$
 (B) $a_k = 0$, for k even integer and $T = 3$ (D) $a_k = 0$, for k odd integer and $T = 6$

[Ans. B]

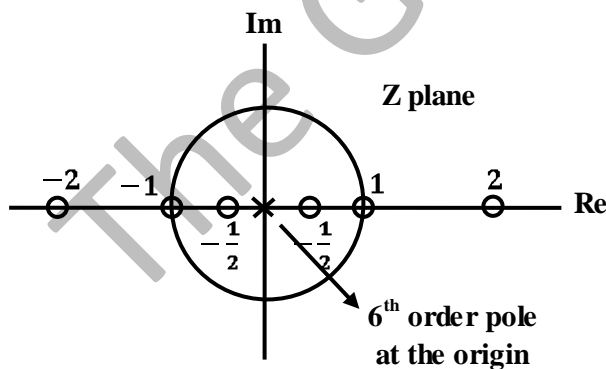
6. The integral

$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} t^2 e^{-\frac{t^2}{2}} \delta(t - 2t) dt$ is equal to

- (A) $\frac{1}{8\sqrt{2\pi}} e^{-1/8}$ (C) $\frac{1}{\sqrt{2\pi}} e^{-1/2}$
 (B) $\frac{1}{4\sqrt{2\pi}} e^{-1/8}$ (D) 1

[Ans. D]

7. Shown below is the pole-zero plot of a digital filter



Which one of the following statements is **TRUE**?

- (A) This is a low pass filter (C) This is an IIR filter
 (B) This is a high pass filter (D) This is an FIR filter

[Ans. D]

$$H(z) = \left(1 - \frac{1}{4}z^{-2}\right)(1 - z^{-2})(1 - 4z^{-2})$$

\therefore Filter is FIR

8. The continuous time signal $x(t) = \cos(100\pi t) + \sin(300\pi t)$ is sampled at the rate 100 Hz to get the signal

$$x_g(t) = \sum_{n=-\infty}^{\infty} x(nT_g)\delta(t - nT_g), T_g = \text{sampling period}$$

The signal $x_g(t)$ is passed through an ideal low pass filter with cutoff frequency 100 Hz. The output of the filter is proportional to

- (A) $\cos(100 \pi t)$ (C) $\cos(100 \pi t) - \sin(100 \pi t)$
 (B) $\cos(100 \pi t) + \sin(100 \pi t)$ (D) $\sin(100 \pi t)$

[Ans. B]

$\cos(100 \pi t)$ doesn't contribute to aliasing.

$$\sin(300 \pi t) \text{ leads to aliasing } \Rightarrow \sin((300 \pi - 200 \pi)t) = \sin(100 \pi t)$$

9. Consider a system with input $x(t)$ and output $y(t)$ related as follows

$$y(t) = \frac{d}{dt} \{e^{-t}x(t)\}$$

Which one of the following statements is TRUE?

- (A) The system is nonlinear (C) The system is stable
 (B) The system is time-invariant (D) The system has memory

[Ans. C]

System is linear, time-variant, memoryless and stable.

10. The first two rows of Routh's table of a third-order characteristic equation are

$$\begin{array}{ccc} s^3 & 3 & 3 \\ s^2 & 4 & 4 \end{array}$$

It can be inferred that the system has

- (A) one real pole in the right-half of s-plane
 (B) a pair of complex conjugate poles in the right-half of s-plane
 (C) a pair of real poles symmetrically placed around $s = 0$
 (D) a pair of complex conjugate poles on the imaginary axis of the s-plane

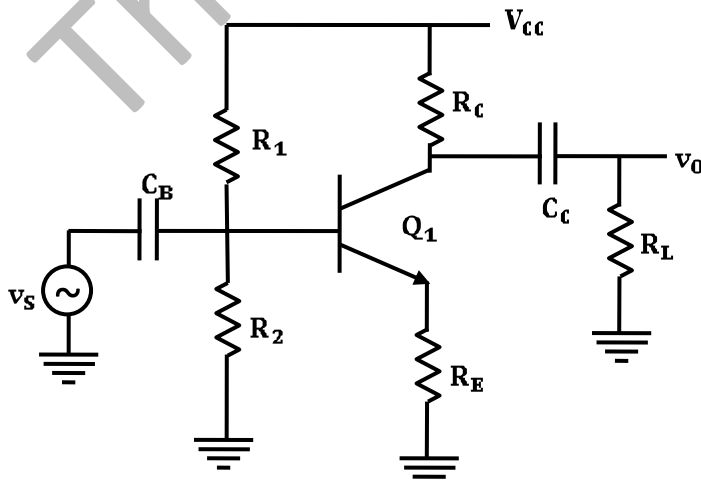
[Ans. D]

$$\begin{array}{ccc} s^3 & 3 & 3 \\ s^2 & 4 & 4 \\ S & 0(\epsilon > 0) & \\ 1 & 4 & \end{array}$$

$$\therefore \text{C.E is } 3S^3 + 4S^2 + 3S + 4 = 0 \Rightarrow (3S + 4)(S^2 + 1) = 0$$

$$S = -\frac{4}{3}, \pm j$$

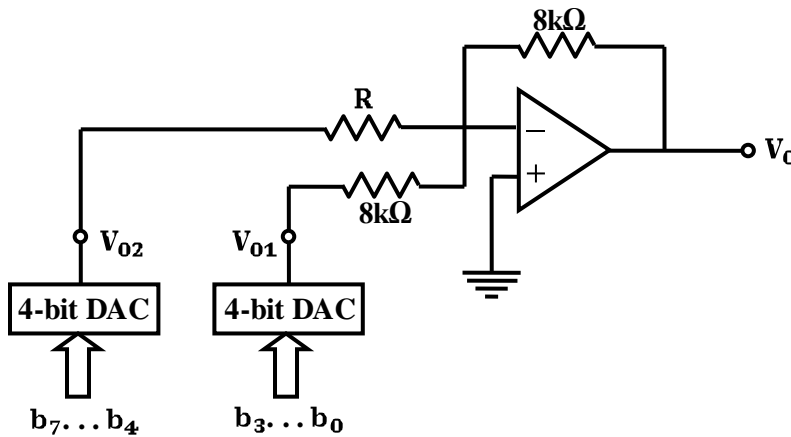
11. The amplifier shown below has a voltage gain of -2.5 , an input resistance of $10 \text{ k}\Omega$ and a lower 3-dB cut-off frequency of 20 Hz. Which one of the following statements is **TRUE** when the emitter resistance R_E is doubled?



- (A) Magnitude of voltage gain will decrease (C) Collector bias current will increase
 (B) Input resistance will decrease (D) Lower 3-dB cut-off frequency will increase

[Ans. A]

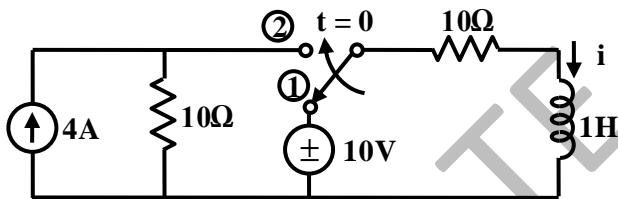
12. Figure below shows a circuit for implementing an 8-bit Digital-to-Analog converter (DAC) using two identical 4-bit DACs with equal reference voltages. Assume that b_0 represents LSB, b_7 MSB an the opamp is ideal. To obtain correct analog values corresponding to an 8-bit DAC at the output V_0 , the value of resistor R is



- (A) 0.25 kΩ
(B) 0.5 kΩ
(C) 1 kΩ
(D) 8 kΩ

[Ans. B]

13. In the circuit shown below, the switch, initially at position 1 for a long time, is changed to position 2 at $t = 0$.



The current i through the inductor for $t \geq 0$ is

- (A) $1 - e^{-20t}$ A
(B) $1 + e^{-20t}$ A
(C) $1 + 2e^{-20t}$ A
(D) $2 - e^{-20t}$ A

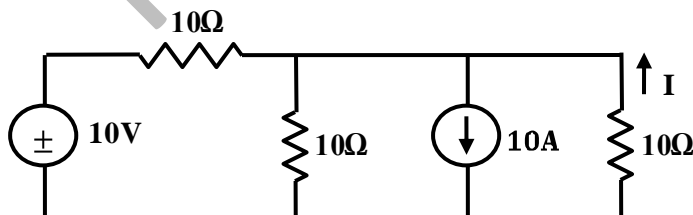
[Ans. D]

$$i(0^-) = 1A$$

$$\text{For } t \geq 0, I_s = 1A; I_f = 2A; \tau = \frac{1}{20} \text{ sec}$$

$$\therefore i(t) = (2 - e^{-20t})u(t) \text{ A}$$

14. The current I shown in the circuit given below is equal to



- (A) 3A
(B) 3.67A
(C) 6A
(D) 9A

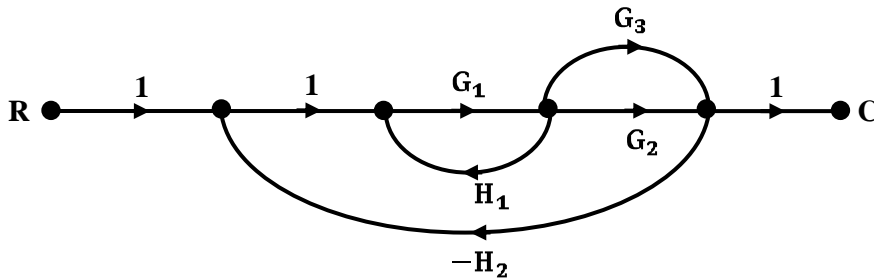
[Ans. A]

By nodal analysis,

$$\frac{(v-10)}{10} + \frac{v}{10} + 10 + \frac{v}{10} = 0 \Rightarrow \frac{3v}{10} = -9 \Rightarrow v = -30V$$

$$\therefore I = \frac{-v}{10} = 3A$$

15. The signal flow graph of a system is given below.



The transfer function (C/R) of the system is

(A) $\frac{(G_1 G_2 + G_1 G_3)}{(1 + G_1 G_2 H_2)}$

(C) $\frac{(G_1 G_2 + G_1 G_3)}{(1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_3 H_2)}$

(B) $\frac{(G_1 G_2 + G_1 G_3)}{(1 + G_1 H_1 + G_1 G_2 H_2)}$

(D) $\frac{(G_1 G_2 + G_1 G_3)}{(1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_3 H_2 + G_1 G_2 G_3 H_1)}$

[Ans. C]

$P_1 = G_1 G_2; P_2 = G_1 G_3$

$L_1 = G_1 H_1; L_2 = -G_1 G_2 H_2; L_3 = -G_1 G_3 H_2$

$\Delta_1 = 1; \Delta_2 = 1$

$\Delta = 1 - (L_1 + L_2 + L_3) = 1 - G_1 H_1 + G_1 G_2 H_2 + G_1 G_3 H_2$

$\therefore \frac{C}{R} = \frac{(P_1 \Delta_1 + P_2 \Delta_2)}{\Delta} = \frac{(G_1 G_2 + G_1 G_3)}{(1 - G_1 H_1 + G_1 G_2 H_2 + G_1 G_3 H_2)}$

16. For the Boolean expression $f = \bar{a}\bar{b}\bar{c} + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc + ab\bar{c}$, the minimized Product of Sum (PoS) expression is

(A) $f = (b + \bar{c}).(a + \bar{c})$

(C) $f = (\bar{b} + c).(a + \bar{c})$

(B) $f = (\bar{b} + c).(\bar{a} + c)$

(D) $f = \bar{c} + abc$

[Ans. A]

17. The base of the number system for the addition operation $24 + 14 = 41$ to be true is

(A) 8

(C) 6

(B) 7

(D) 5

[Ans. B]

18. An $8K \times 8$ bit RAM is interfaced to an 8085 microprocessor. In a fully decoded scheme if the address of the last memory location of this RAM is 4FFFH, the address of the first memory location of the RAM will be

(A) 1000 H

(C) 3000 H

(B) 2000 H

(D) 4000 H

[Ans. C]

19. The Treadmill Test is used to diagnose

(A) the balancing style during walk of the patient

(B) the auditory activity of the patient

(C) the visual activity of the patient

(D) the cardiac activity of the patient

[Ans. D]

Treadmill test is generally recommended for patients with coronary artery blockages. Hence it is used to diagnose the cardiac activity of patient. It is some what similar to what our ECG do.

20. The characteristics of a thermometer measuring ambient temperature is $2 \frac{dT_i}{dt} + T_i - T_a = 0$, where T_i and T_a are the indicated and ambient temperatures, respectively, both in $^{\circ}\text{C}$ and time is in seconds. The -3 dB cut-off frequency in the frequency response of the thermometer is

- (A) $\frac{1}{4\pi}$ Hz (C) 1 Hz
(B) $\frac{1}{4}$ Hz (D) 2π Hz

[Ans. A]

$$\frac{T_a(s)}{T_i(s)} = \frac{1}{(2s+1)} \Rightarrow G(j\omega) = \frac{1}{(2j+1)}$$

$$|G(j\omega)| = \frac{1}{\sqrt{1+4\omega^2}} = \frac{1}{\sqrt{2}} \Rightarrow \omega = \frac{1}{2} \Rightarrow f = \frac{1}{4\pi} \text{ Hz}$$

21. For a copper-constantan (Type T) thermocouple, the junction potential E (in μV) at $\theta^{\circ}\text{C}$ is given by $E = 38.74\theta + 3.3 \times 10^{-2}\theta^2 + 2.07 \times 10^{-4}\theta^3 - 2.2 \times 10^{-6}\theta^4 +$ higher order terms, assuming the cold junction compensation. The sensitivity of thermocouple at 100°C is approximately

- (A) $45.34 \mu\text{V}/^{\circ}\text{C}$ (C) $38.74 \mu\text{V}/^{\circ}\text{C}$
(B) $42.75 \mu\text{V}/^{\circ}\text{C}$ (D) $0.06 \mu\text{V}/^{\circ}\text{C}$

[Ans. B]

22. The temperature of a furnace is monitored at a distance of 50 m away. The temperature transmitter has a range of $0 - 500^{\circ}\text{C}$ and provides $4-20$ mA current output. The measured temperature and the output current have a straight line relationship with positive slope. The temperature is determined from the voltage measured across a resistance of 500Ω in the current loop. If the voltage measured across the resistance is 4V , the temperature of the furnace is

- (A) 100°C (C) 150°C
(B) 125°C (D) 200°C

[Ans. B]

23. The core/cladding index difference of a single-mode optical fiber cable is 0.01 . The refractive index of the material of the core is 1.5 . The maximum angle of acceptance of the fiber is approximately equal to

- (A) 17.5° (C) 8.6°
(B) 12.1° (D) 2.0°

[Ans. B]

Given:- Core/cladding index difference = 0.01

$$\Delta = 0.01$$

Refractive index of core, $n_1 = 1.5$

Numerical Aperture we will calculate

$$N.A = n_1 \times \sqrt{2\Delta}$$

$$N.A = 1.5 \times \sqrt{2 \times 0.01}$$

$$N.A = 1.5 \times \sqrt{0.02}$$

$$N.A = 0.212$$

So, acceptance angle can be given by

$$\theta_m = \sin^{-1}(N.A)$$

$$\theta_m = \sin^{-1}(0.212)$$

$$\theta_m = 12.24 \approx 12.1^{\circ}$$

$$\theta_m = 12.1^{\circ}$$

24. The conventional way of expressing vibration is in terms of

- (A) Richter scale (C) speed of sound
(B) acceleration due to gravity (D) atmospheric pressure

[Ans. B]

25. The primary and secondary of an LVDT (stroke length ± 50 mm) are connected to a 3 kHz sinusoidal source and ideal semiconductor diode bridge-based phase sensitive demodulator circuit. The core of the LVDT remains static at 15 mm above the ideal null position. The frequency of the voltage observed at the input of the low-pass filter is
- (A) 1 kHz (C) 3 kHz
(B) 1.5 kHz (D) 6 kHz

[Ans. C]

Q. 26 to Q.55 Carry Two Marks each.

26. The series $\sum_{m=0}^{\infty} \frac{1}{4^m} (x-1)^{2m}$ converges for
- (A) $-2 < x < 2$ (C) $-3 < x < 1$
(B) $-1 < x < 3$ (D) $x < 3$

[Ans. B]

In a G.P $r = \frac{1}{4} (x-1)^2$

For a G.P to converge $-1 < r < 1$

$$\Rightarrow \frac{(x-1)^2}{4} < 1 \Rightarrow -2 < (x-1) < 2$$

$$\Rightarrow -1 < x < 3$$

27. Consider the differential equation $\ddot{y} + 2\dot{y} + y = 0$ with boundary conditions $y(0) = 1, y(1) = 0$. The value of $y(2)$ is
- (A) -1 (C) $-e^{-2}$
(B) $-e^{-1}$ (D) $-e^2$

[Ans. C]

$$D^2 + 2D + 1 = 0$$

$$D = -1, -1$$

The solution for the differential equation is

$$y = (C_1 + C_2 x) e^{-x}$$

Now, $y(0) = 1$ and $y(1) = 0$, placing these values

We get, $C_1 = 1$ and $C_2 = -1$

$$\therefore y(2) = (1 + (-1)2) e^{-2} = -e^{-2}$$

28. The box 1 contains chips numbered 3, 6, 9, 12 and 15. The box 2 contains chips numbered 6, 11, 16, 21 and 26. Two chips, one from each box, are drawn at random. The numbers written on these chips are multiplied. The probability for the product to be an even number is
- (A) $\frac{6}{25}$ (C) $\frac{3}{5}$
(B) $\frac{2}{5}$ (D) $\frac{19}{25}$

[Ans. D]

For the product to be even, the numbers from both the boxes should not turn out to be odd simultaneously.

$$\therefore P(E) = 1 - \left(\frac{3}{5}\right) \left(\frac{2}{5}\right) = \frac{19}{25}$$

29. The extremum (minimum or maximum) point of a function $f(x)$ is to be determined by solving $\frac{df(x)}{dx} = 0$ using the Newton-Raphson method. Let $f(x) = x^3 - 6x$ and $x_0 = 1$ be the initial guess of x . The value of x after two iterations (x_2) is
- (A) 0.0141 (C) 1.4167
(B) 1.4142 (D) 1.5000

[Ans. C]

$$f(x) = x^3 - 6x$$

$$f'(x) = 3x^2 - 6 = g(x)$$

$x_0 = 1$, initial guess

$$g'(x) = 6x$$

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)}$$

$$= 1 - \frac{(-3)}{6} = 1.5$$

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)}$$

$$= 1.5 - \frac{0.75}{9}$$

$$= 1.4167$$

30. The unit-step response of a negative unity feedback system with the open-loop transfer function $G(s) = \frac{6}{s+5}$

is

(A) $1 - e^{-5t}$

(B) $6 - 6e^{-5t}$

(C) $\frac{6}{5} - \frac{6}{5}e^{-5t}$

(D) $\frac{6}{11} - \frac{6}{11}e^{-11t}$

[Ans. D]

$$T(s) = \frac{6}{(s+1)} \Rightarrow Y_u(s) = \frac{6}{s(s+1)} = \frac{6}{11s} + \frac{-6}{11(s+1)}$$

$$\therefore Y_u(t) = \frac{6}{11}(1 - e^{-11t})u(t)$$

31. The transfer function of the system described by the state-space equations

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -4 & -1 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u, \quad y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$
 is

(A) $\frac{s}{s^2+5s+1}$

(B) $\frac{2s}{s^2+5s+1}$

(C) $\frac{3s}{s^2+5s+1}$

(D) $\frac{4s}{s^2+5s+1}$

[Ans. A]

$$T(s) = C(SI - A)^{-1}B + D = [1 \quad 0] \frac{1}{(s^2+5s+1)} \begin{bmatrix} s+1 & -1 \\ -3 & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{s}{(s^2+5s+1)}$$

32. Consider the second-order system with the characteristic equation $s(s+3) + K(s+5) = 0$. Based on the properties of the root loci, it can be shown that the complex portion of the root loci of the given system for $0 < K < \infty$ is described by a circle, and the two breakaway points on the real axis are

(A) $-5 \pm \frac{\sqrt{5}}{2}$

(B) $-5 \pm \sqrt{5}$

(C) $-5 \pm \sqrt{10}$

(D) $-5 \pm 2\sqrt{5}$

[Ans. C]

$$K = \frac{-s(s+3)}{(s+5)}$$

$$\frac{dk}{ds} = 0 \Rightarrow \frac{-((s+5)(2s+3) - (s^2+3s) \cdot 1)}{(s+5)^2} = 0 \Rightarrow s^2 + 10s + 15 = 0$$

$$s = \frac{-10 \pm \sqrt{100-60}}{2} = -5 \pm \sqrt{10}$$

33. In a flapper-nozzle displacement transducer, the values of the following parameters are given:

Diameter of the orifice = 0.2 mm, Diameter of the nozzle = 0.8 mm,

Supply pressure = 1.4×10^2 kPa (gauge), Ambient pressure = 0 (gauge).

The maximum value of the sensitivity is

(A) 4.0 MPa/mm

(B) 5.6 MPa/mm

(C) 6.4 MPa/mm

(D) 7.3 MPa/mm

[Ans. B]

34. A differential push-pull type capacitive displacement sensor (nominal capacitance $C_0 = 0.01\mu\text{F}$) is connected in two adjacent arms of an ac bridge in such a way that the output voltage of the bridge is independent of the frequency of the supply voltage. Supply to the bridge is 1V at 1kHz, and two equal resistances ($R = 3.9\text{ k}\Omega$) are placed in the other two arms of the bridge. The bridge sensitivity is

(A) 0.001 mV/pF

(C) 0.1 mV/pF

(B) 0.05 mV/pF

(D) 0.5 mV/pF

[Ans. *]

35. A turbine flowmeter is rotating at 72 rpm. The flux Ψ linked to the nearby magnet and coil assembly is given by $\Psi(\theta) = 3 + \cos(4\theta)$ mWb, where θ is the angular position (in radian). The amplitude and frequency of the output voltage signal, respectively, are

(A) 4 mV and 45.8 Hz

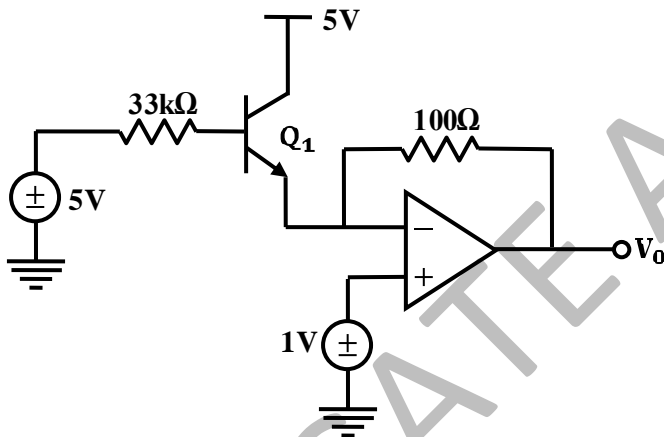
(C) 30.2 mV and 30.2 Hz

(B) 30.2 mV and 4.8 Hz

(D) 288 mV and 45.8 Hz

[Ans. *]

36. Assuming base-emitter voltage of 0.7V and $\beta = 99$ of transistor Q_1 , the output voltage V_0 in the ideal opamp circuit shown below is



(A) -1V

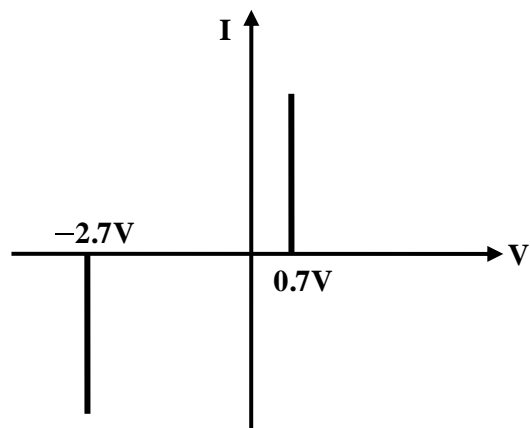
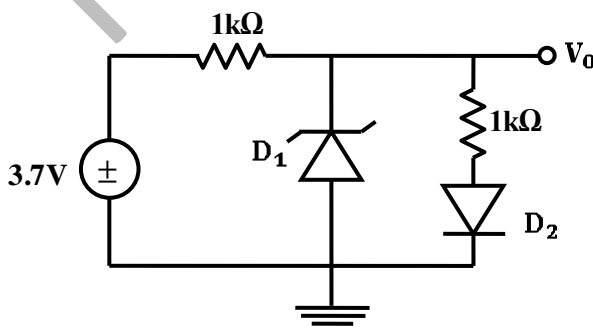
(C) 0V

(B) -1/3.3V

(D) 2V

[Ans. C]

37. Assuming zener diode D_1 has current-voltage characteristics as shown below on the right and forward voltage drop of diode D_2 is 0.7 V, the voltage V_0 in the circuit shown below is



(A) 3.7 V

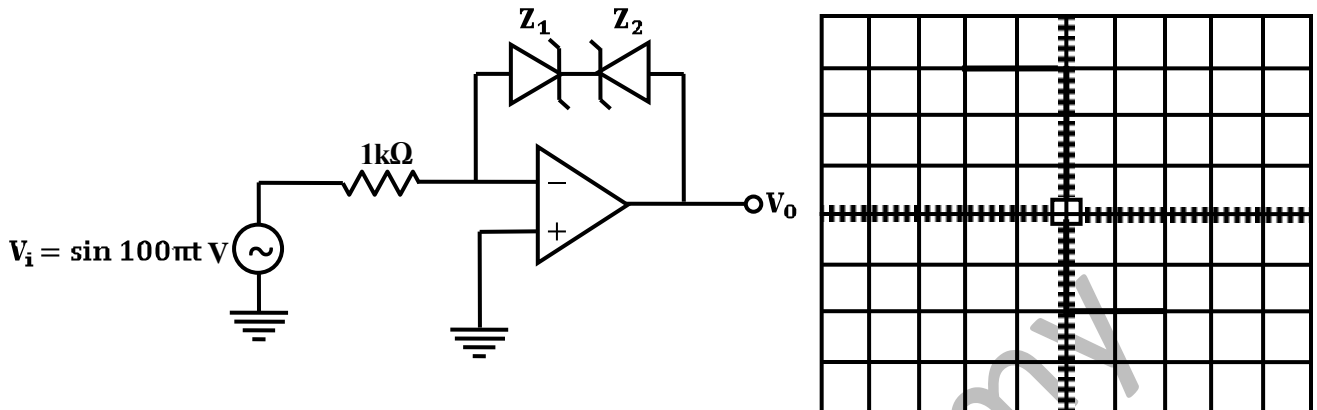
(C) 2.2 V

(B) 2.7 V

(D) 0 V

[Ans. C]

38. The transfer characteristics of the circuit drawn below is observed on an oscilloscope used in XY mode. The display on the oscilloscope is shown on the right hand side. V_t is connected to the X input with a setting of 0.5 V/div, and V_o is connected to the Y input with a setting of 2 V/div. The beam is positioned at the origin when V_i is zero.



Assuming that the opamp is ideal and the zener diodes have forward biased voltage drop of 0.7V, the values of reverse break-down voltages of Z_1 and Z_2 are, respectively.

- (A) 3.3 V and 5.3 V
(B) 4.7 V and 6.7 V
(C) 6.7 V and 4.7 V
(D) 5.3 V and 3.3 V

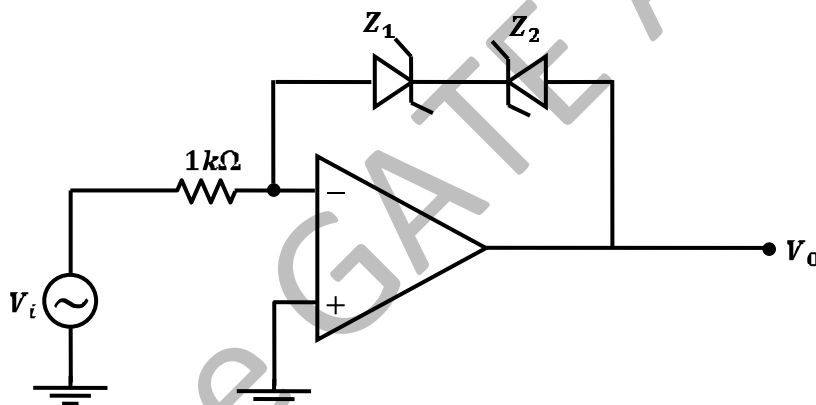
[Ans. D]

$$V_o = -(V_1 + V_2)$$

During positive cycle

$$V_o = 6V \text{ and } V_2 \text{ is F-B}$$

$$\therefore V_1 = 6 + 0.7 = 6.7V$$



During negative cycle

$$4 + 0.7 = V_2$$

$$V_2 = 4.7 V$$

39. Power in a three phase star connected balanced inductive load is measured by two wattmeter method. The phase voltage and phase current are 230 V and 5 A, respectively. The power factor of the load is 0.707. The reading P_1 and P_2 of the two wattmeters are

- (A) $P_1 = 298 \text{ W}$ and $P_2 = 1111 \text{ W}$
(B) $P_1 = 516 \text{ W}$ and $P_2 = 1924 \text{ W}$
(C) $P_1 = 1220 \text{ W}$ and $P_2 = 1220 \text{ W}$
(D) $P_1 = 1111 \text{ W}$ and $P_2 = -516 \text{ W}$

[Ans. B]

For a balanced lagging load, connected in a star network.

The two wattmeter readings are:

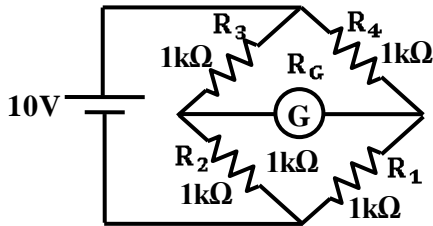
$$w_2 = V_L I_L \cos(30 - \phi)$$

$$w_1 = V_L I_L \cos(30 + \phi)$$

Given the load is balanced, hence line voltage is equal to phase voltage. Similarly for line and phase currents.

Given, $\cos \phi = 0.707 \rightarrow \phi = 45^\circ$
 $w_2 = 230 \times 5 \cos(30 - 45)$
 $w_2 = 1111w$
 $w_1 = V_L I_L \cos(30 + \phi)$
 $= 230 \times 5 \times \cos(75^\circ)$
 $w_1 = 298w$

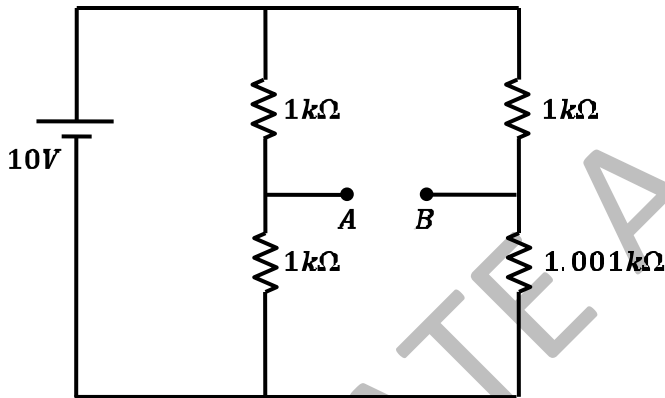
40. In the Wheatstone bridge shown below, when the resistance R_1 increases by 1Ω , the current through the galvanometer is



(consider the Thevenin equivalent resistance of the bridge in the calculations)

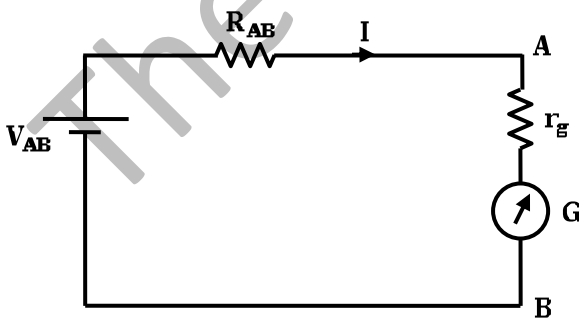
- (A) $1.25 \mu A$ (C) $12.5 \mu A$
 (B) $2.5 \mu A$ (D) $25 \mu A$

[Ans. A]



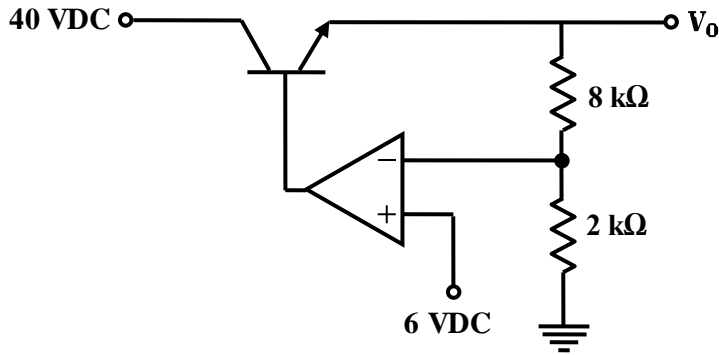
Thevenin ead circuit across AB.

$V_{AB} = V_A - V_B = (5 - 5.0025) V$
 $= 2.498mV$
 $R_{AB} = (1K // 1K) + (1K // 1.001K)$
 $= 1.00025K\Omega$



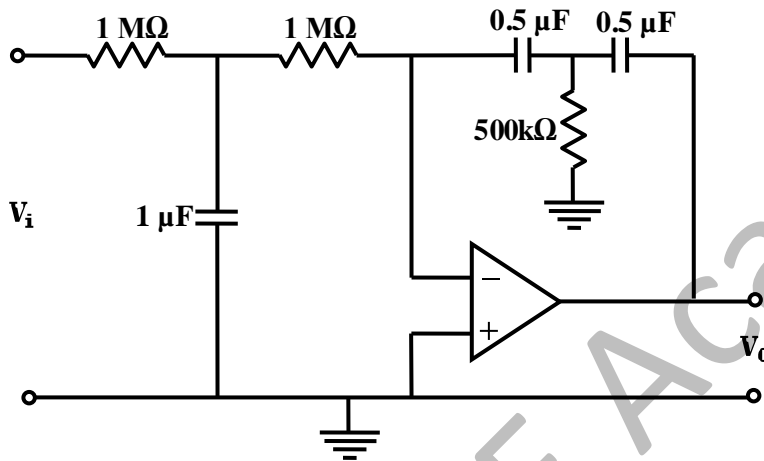
$I = \frac{V_{AB}}{R_{AB} + r_g} = \frac{2.498 mV}{1.00025K\Omega}$
 $= 1.25 \mu A$

41. The value of V_0 of the series regulator shown below is



- (A) 24 V
(B) 28 V
(C) 30 V
(D) 32 V
- [Ans. C]

42. The ideal opamp based circuit shown below acts as a



- (A) low-pass filter
(B) high-pass filter
(C) band-pass filter
(D) band-reject filter
- [Ans. C]

43. A 4-bit successive approximation type of A/D converter has an input range of 0 to 15 volts. The output bit b_1 next to the LSB has a stuck-at-zero fault. The pair of input voltages that produces the same output code word is

- (A) 2V and 4V
(B) 4V and 6V
(C) 1V and 2V
(D) 8V and 9V

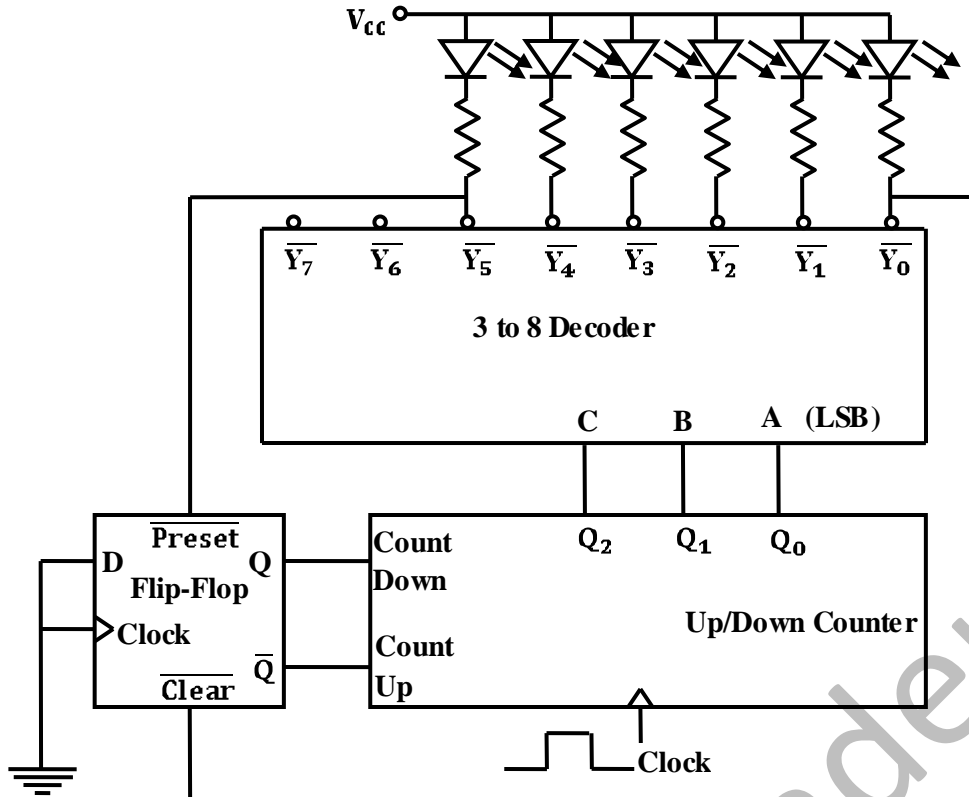
[Ans. B]

44. The number of objects crossing a window sequentially at variable speed is to be counted using an interrupt in the 8085 microprocessor. The objects are sensed by an optical source and a detector with associated signal conditioning circuit. The circuit produces a logic high output as long as the object is in front of the window, and this output is used to interrupt the processor. The duration of an object being in front of the window is in the range of 100 ms to 2s. The processor takes 1 ms to process the input after an interrupt. The best choice of interrupt for an error fee counting is

- (A) RST 5.5
(B) RST 6.5
(C) RST 7.5
(D) INTR

[Ans. D]

45. The circuit below shows an up/down counter working with a decoder and a flip-flop. Preset and Clear of the flip-flop are asynchronous active-low inputs.



Assuming that the initial value of counter output ($Q_2 Q_1 Q_0$) as zero, the counter outputs in decimal for 12 clock cycles are

- (A) 0, 1, 2, 3, 4, 4, 3, 2, 1, 1, 2, 3, 4
 (B) 0, 1, 2, 3, 4, 5, 0, 1, 2, 3, 4, 5, 0
 (C) 0, 1, 2, 3, 4, 5, 5, 4, 3, 2, 1, 0, 1
 (D) 0, 1, 2, 3, 4, 5, 4, 3, 2, 1, 0, 1, 2

[Ans. D]

46. A square wave (amplitude ± 10 mV, frequency 5 kHz, duty cycle 50%) is passed through an ideal low-pass filter with pass-band gain and cut-off frequency of 0 dB and 10 kHz respectively. The filtered signal is subsequently "buried" additively into a zero-mean noise process of one-sided power-spectral density (PSD) of 25 pW Hz^{-1} up to a frequency of 2 MHz. The PSD of the noise is assumed to be zero beyond 2 MHz. The signal-to-noise ratio of the output is

- (A) 0 dB
 (B) 0.1 dB
 (C) 1.0 dB
 (D) 3 dB

[Ans. *]

47. Consider the difference equation $y[n] - \frac{1}{3}y[n-1] = x[n]$ and suppose that $x[n] = \left(\frac{1}{2}\right)^n u[n]$. Assuming the condition of initial rest, the solution for $y[n]$, $n \geq 0$ is

- (A) $3 \left(\frac{1}{3}\right)^n - 2 \left(\frac{1}{2}\right)^n$
 (B) $-2 \left(\frac{1}{3}\right)^n + 3 \left(\frac{1}{2}\right)^n$
 (C) $\frac{2}{3} \left(\frac{1}{3}\right)^n + \frac{1}{3} \left(\frac{1}{2}\right)^n$
 (D) $\frac{1}{3} \left(\frac{1}{3}\right)^n + \frac{2}{3} \left(\frac{1}{2}\right)^n$

[Ans. B]

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{(1 - \frac{1}{3}z^{-1})}; X(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})}$$

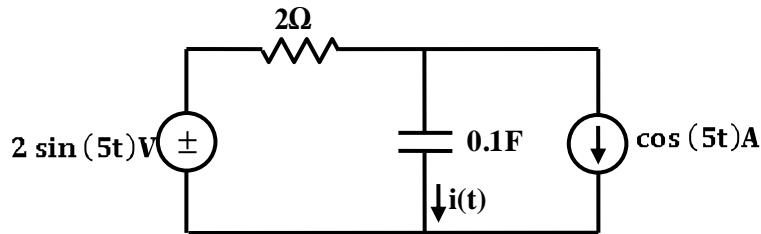
$$\therefore Y(z) = \frac{1}{(1 - \frac{1}{3}z^{-1})(1 - \frac{1}{2}z^{-1})} = \frac{-2}{(1 - \frac{1}{3}z^{-1})} + \frac{3}{(1 - \frac{1}{2}z^{-1})}$$

$$\therefore y[n] = \left(-2 \left(\frac{1}{3}\right)^n + 3 \left(\frac{1}{2}\right)^n\right) u[n]$$

Common Data Questions

Common Data for Questions 48 and 49:

Consider the circuit shown below.



48. The current $i(t)$ = through the capacitor is

- (A) $\sin(5t)A$ (C) $\sin(5t - 45^\circ)A$
 (B) $\cos(5t)A$ (D) $1A$

[Ans. B]

By nodal analysis

$$\frac{(\underline{V} - 2\angle 0^\circ)}{2} + \frac{\underline{V}}{(-2j)} + 1\angle -\frac{\pi}{2} = 0$$

$$\Rightarrow \underline{V}(0.5 + 0.5j) = (1 + j) \Rightarrow \underline{V} = \frac{\sqrt{2}\angle +\frac{\pi}{4}}{0.5\sqrt{2}\angle \frac{\pi}{4}} = 2\angle 0^\circ$$

$$\underline{I} = \frac{\underline{V}}{-2j} = 1\angle \frac{\pi}{2}$$

$$\therefore i(t) = \cos(5t)A$$

49. The average total power delivered by the two sources is

- (A) 0 W (C) 2 W
 (B) 0.5 W (D) 4 W

[Ans. A]

$$P_S = P_{S_1} + P_{S_2} = P_2 = i_2^2 \times 2$$

$$i_2 = \frac{(\underline{V} - 2\angle 0^\circ)}{2} = 0$$

$$\therefore P_S = 0w$$

Common Data for Questions 50 and 51:

The open-loop transfer function of a unity negative feedback control system is given by

$$G(s) = \frac{K}{(s+5)^2}$$

50. The value of K for the phase margin of the system to be 45° is

- (A) $250\sqrt{5}$ (C) $125\sqrt{5}$
 (B) $250\sqrt{2}$ (D) $125\sqrt{2}$

[Ans. B]

$$PM = 45^\circ = 180^\circ - 3 \tan^{-1}\left(\frac{5}{\omega}\right) = 45^\circ \Rightarrow \tan^{-1}\left(\frac{5}{\omega}\right) = 1 \Rightarrow \omega = 5 \text{ rad/sec}$$

$$\therefore |G(j\omega)| = 1 \Rightarrow \left| \frac{K}{(5+5j)^2} \right| = 1 \Rightarrow K = 250\sqrt{2}$$

51. The value of K for the damping ratio ζ to be 0.5, corresponding to the dominant closed-loop complex conjugate pole pair is

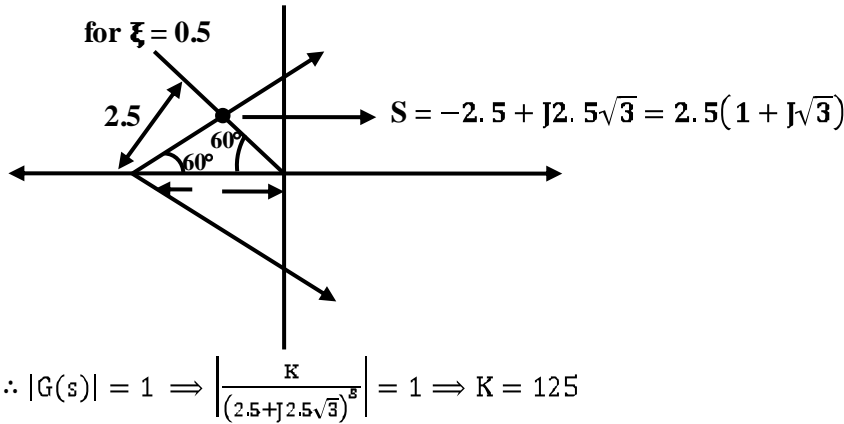
- (A) 250 (C) 75
 (B) 125 (D) 50

[Ans. B]

$$\xi = 0.5 \Rightarrow \theta = \cos^{-1}(\xi) = 60^\circ$$

Find K at which 60° intersects with RL

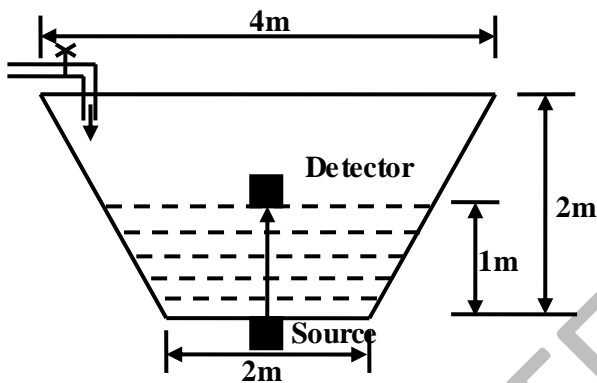
RL plot is as shown below



Linked Answer Questions

Statement for Linked Answer Questions 52 and 53:

The level of water, stored in a truncated conical bath, is measured by a gamma-ray radiation sensor. The initial level of water is 1 m, and the level is increasing due to water inflow at the constant rate of $0.125 \text{ m}^3/\text{s}$. Assume mass absorption coefficient of water is $77 \times 10^{-4} \text{ m}^2/\text{kg}$ and density of water is 1000 kg/m^3 .

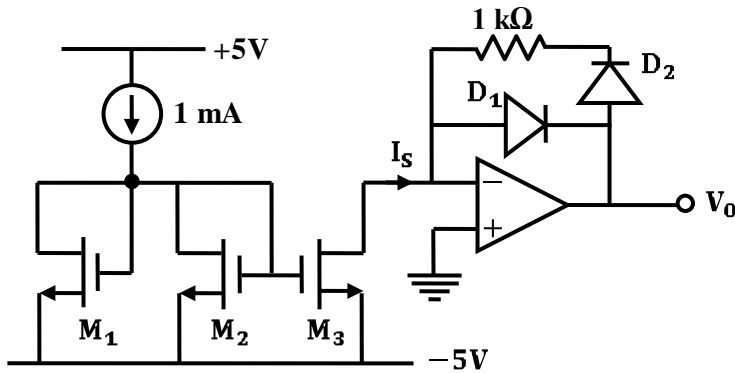


(Figure is not to scale)

52. When the intensity of radiation received by the floating detector is half of the intensity detected initially, the level of water is
- (A) 1.09 m (C) 1.8 m
(B) 1.5 m (D) 1.9 m
- [Ans. A]
53. When the floating detector is at the level calculated in Q.52, the time elapsed is
- (A) 4.1 s (C) 10.52 s
(B) 5.23 s (D) 50.63 s
- [Ans. A]

Statement for Linked Questions 54 and 55:

M1, M2 and M3 in the circuit shown below are matched N-channel enhancement mode MOSFETs operating in saturation mode, forward voltage drop of each diode is 0.7 V, reverse leakage current of each diode is negligible and the opamp is ideal.



54. The current I_S in the circuit is
 (A) -1 mA (C) 1 mA
 (B) 0.5 mA (D) 2 mA

[Ans. B]

55. For the computed value of current I_S , the output voltage V_O is
 (A) 1.2 V (C) 0.2 V
 (B) 0.7 V (D) -0.7 V

[Ans. A]

General Aptitude (GA) Questions

Q.56 – Q. 60 carry one mark each.

56. There are two candidates P and Q in an election. During the campaign, 40% of the voters promised to vote for P, and rest for Q. However, on the day of election 15% of the voters went back on their promise to vote for P and instead voted for Q. 25% of the voters went back on their promise to vote for Q and instead voted for P. Suppose, P lost by 2 votes, then what was the total number of voters?
 (A) 100 (C) 90
 (B) 110 (D) 95

[Ans. A]

57. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair:

Gladiator : Arena

- (A) dancer : stage (C) teacher : classroom
 (B) commuter : train (D) lawyer : courtroom

[Ans. D]

58. Choose the most appropriate word from the options given below to complete the following sentence:
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which _____ treatments are unsatisfactory.

- (A) similar (C) uncommon
 (B) most (D) available

[Ans. D]

59. Choose the word from the options given below that is most nearly opposite in meaning to the given word:

Frequency

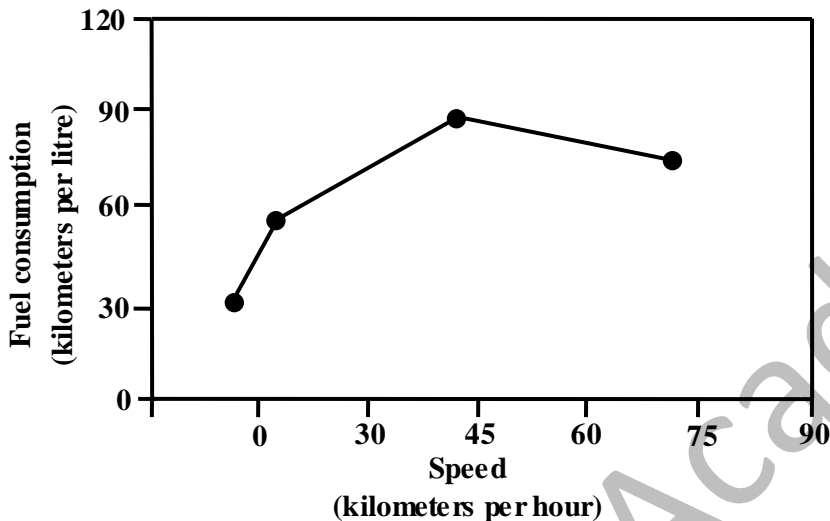
- (A) periodicity (C) gradualness
 (B) rarity (D) persistency

[Ans. A]

60. Choose the most appropriate word from the options given below to complete the following sentence:
It was her view that the country's problems had been _____ by foreign technocrats, so that to invite them to come back would be counter-productive.
 (A) identified (C) exacerbated
 (B) ascertained (D) analysed
[Ans. D]

Q. 61 to Q. 65 carry two marks each.

61. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below.



The distances covered during four laps of the journey are listed in the table below

Lap	Distance (Kilometres)	Average speed (Kilometres per hour)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data we can conclude that the fuel consumed per kilometre was least during the lap

- (A) P (C) R
 (B) Q (D) S
[Ans. C]
62. **The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.**
 It can be inferred from the passage, that horses were
 (A) given immunity to diseases (C) given medicines to fight toxins
 (B) generally quite immune to diseases (D) given diphtheria and tetanus serums
[Ans. B]
63. The sum of n terms of series $4 + 44 + 444 + \dots$ is
 (A) $(4/81)[10^{n+1} - 9n - 1]$ (C) $(4/81)[10^{n+1} - 9n - 10]$
 (B) $(4/81)[10^{n-1} - 9n - 1]$ (D) $(4/81)[10^n - 9n - 10]$
[Ans. C]

64. Given that $f(y) = |y|/y$, and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is
 (A) 0 (B) -1

(C) 1

(D) 2

[Ans. D]

65. Three friends, R, S and T shared toffee from a bowl. R took $1/3^{\text{rd}}$ of the toffees, but returned four to the bowl. S took $1/4^{\text{th}}$ of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?

(A) 38

(C) 48

(B) 31

(D) 41

[Ans. C]

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