

## GATE 2011

### Computer Science Engineering

Set – C

**Q.1 - Q.25 Carry One Mark each.**

1. Consider a relational table with a single record for each registered student with the following attributes.
1. Registration\_Num: Unique registration number of each registered student
  2. UID: Unique identity number, unique at the national level for each citizen
  3. BankAccount\_Num: Unique account number at the bank. A student can have multiple
  4. Name: Name of student
  5. Hostel\_Room: Room number of the hostel

Which of the following options is **INCORRECT**?

- (A) BankAccount\_Num is a candidate key
- (B) Registration\_Num can be a primary key
- (C) UID is a candidate key if all students are from the same country
- (D) If S is a superkey such that  $S \cap \text{UID}$  is NULL then  $S \cup \text{UID}$  is also a superkey

**[Ans. A]**

Since students can have joint accounts, two students can have same bank account number. So, bank account number can not be candidate key.

2. A computer handles several interrupt sources of which of the following are relevant for this question.
- Interrupt from CPU temperature sensor (raises interrupt if CPU temperature is too high)
  - Interrupt from Mouse (raises interrupt if the mouse is moved or a button is pressed)
  - Interrupt from Keyboard (raises interrupt when a key is pressed or released)
  - Interrupt from Hard Disk (raises interrupt when a disk read is completed)

Which one of these will be handled at the **HIGHEST** priority?

- (A) Interrupt from Hard Disk
- (B) Interrupt from Mouse
- (C) Interrupt from Keyboard
- (D) Interrupt from CPU temperature sensor

**[Ans. D]**

3. Which one of the following is **NOT** desired in a good Software Requirement Specifications (SRS) document?

- (A) Functional Requirements
- (B) Non-Functional Requirements
- (C) Goals of Implementation
- (D) Algorithms for Software Implementation

**[Ans. D]**

Algorithms for implementation should not be part of system requirements.

4. HTML (Hyper Text Markup Language) has language elements which permit certain actions other than describing the structure of the web document. Which one of the following actions is **NOT** supported by pure HTML (without any server or client side scripting) pages?

- (A) Embed web objects from different sites into the same page
- (B) Refresh the page automatically after a specified interval
- (C) Automatically redirect to another page upon download
- (D) Display the client time as part of the page

**[Ans. D]**

Client side scripting is required to display client's system time on page.

5. Which of the following pairs have **DIFFERENT** expressive power?
- (A) Deterministic finite automata (DFA) and Non-deterministic finite automata (NFA)
  - (B) Deterministic push down automata (DPDA) and Non-deterministic push down automata (NPDA)
  - (C) Deterministic single-tape Turing machine and Non-deterministic single-tape Turing machine
  - (D) Single-tape Turing machine and multi-tape Turing machine

[Ans. B]

6. A company needs to develop digital signal processing software for one of its newest inventions. The software is expected to have 40000 lines of code. The company needs to determine the effort in person-months needed to develop this software using the basic COCOMO model. The multiplicative factor for this model is given as 2.8 for the software development on embedded systems while the exponentiation factor is given as 1.20. What is the estimated effort in person-months?

- (A) 234.25
- (B) 932.50
- (C) 287.80
- (D) 122.40

[Ans. A]

KLOC = 40

a = 2.8

b = 1.2

Effort  $a(KLOC^b) = 234.25$

7. Let the time taken to switch between user and kernel modes of execution be  $t_1$  while the time taken to switch between two processes be  $t_2$ . Which of the following is **TRUE**?

- (A)  $t_1 > t_2$
- (B)  $t_1 = t_2$
- (C)  $t_1 < t_2$
- (D) nothing can be said about the relation between  $t_1$  and  $t_2$

[Ans. B]

8. A company needs to develop a strategy for software product development for which it has a choice of two programming languages L1 and L2. The number of lines of code (LOC) developed using L2 is estimated to be twice the LOC developed with L1. The product will have to be maintained for five years. Various parameters for the company are given in the table below.

Parameter	Language L1	Language L2
Man years needed for development	LOC/10000	LOC/10000
Development Cost per man year	Rs. 10,00,000	Rs. 7,50,000
Maintenance time	5 years	5 years
Cost of maintenance per year	Rs. 1,00,000	Rs. 50,000

Total cost of the project includes cost of development and maintenance. What is the LOC for L1 for which the cost of the project using L1 is equal to the cost of the project using L2?

- (A) 4000
- (B) 5000
- (C) 4333
- (D) 4667

[Ans. B]

Let x be the number of lines of code in language L1.

Let  $C_1$  and  $C_2$  be the project costs if we go ahead with L1 and L2 respectively.

$$C_1 = \frac{x}{10000} 1000000 + 5 * 100000 = 100x + 500000$$

$$C_2 = \frac{2x}{10000} 750000 + 5 * 50000 = 150x + 250000$$

Solving  $C_1 = C_2$  gives  $x = 5000$





[Ans. C]

$$p[3] = 'E'$$

$$p[1] = 'A'$$

$$p[3] - p[1] = 4$$

$$p + p[3] - p[1] = p + 4$$

It prints the substring of "GATE2011" starting at index 4.

17. Consider a hypothetical processor with an instruction of type LW R1, 20(R2), which during execution reads a 32-bit word from memory and stores it in a 32-bit register R1. The effective address of the memory location is obtained by the addition of a constant 20 and the contents of register R2. Which of the following best reflects the addressing mode implemented by this instruction for the operand in memory?

(A) Immediate Addressing

(C) Register Indirect Scaled Addressing

(B) Register Addressing

(D) Base Indexed Addressing

[Ans. D]

18. Let the page fault service time be 10 ms in a computer with average memory access time being 20 ns. If one page fault is generated for every  $10^6$  memory access, what is the effective access time for the memory?

(A) 21 ns

(C) 23 ns

(B) 30 ns

(D) 35 ns

[Ans. B]

On an average, for every  $10^6$  memory accesses there is a page fault.

Average time spent during  $10^6$  memory accesses

$$= 10^6 \cdot 20 \text{ ns} + 10 \text{ ms} = 30 \text{ ms}$$

Thus, effective access time per memory access is 30ns.

19. The lexical analysis for a modern computer language such as Java needs the power of which one of the following machine models in a necessary and sufficient sense?

(A) Finite state automata

(C) Non-deterministic pushdown automata

(B) Deterministic pushdown automata

(D) Turing machine

[Ans. A]

Lexical analysis constructs are expressed using regular expressions. So, finite state automata is necessary and sufficient to perform lexical analysis.

20. If the difference between the expectation of the square of a random variable ( $E[X^2]$ ) and the square of the expectation of the random variable ( $E[X]^2$ ) is denoted by R, then

(A)  $R = 0$

(C)  $R \geq 0$

(B)  $R < 0$

(D)  $R > 0$

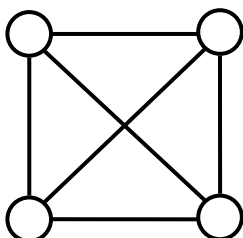
[Ans. C]

$$\text{Variance of } X = E[X^2] - E[X]^2 = E[(X - E[X])^2]$$

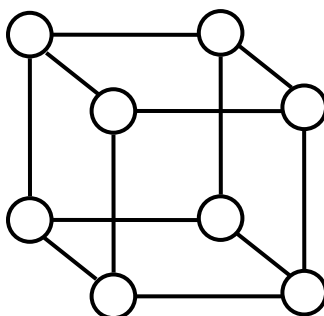
This quantity is always non-negative as it is expectation of a non-negative quantity.

21. K4 and Q3 are graphs with the following structures.

K4



Q3



Which one of the following statements is **TRUE** in relation to these graphs?

- (A) K4 is planar while Q3 is not  
(B) Both K4 and Q3 are planar

- (C) Q3 is planar while K3 is not  
(D) Neither K4 nor Q3 is planar

[Ans. B]

Both graphs are planar.

Here's a planar embedding.

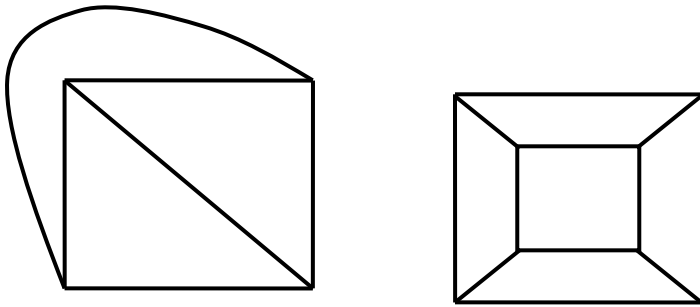


Figure 1: Planar embedding of K4 and Q3

22. A thread is usually defined as a “light weight process” because an operating system (OS) maintains smaller data structures for a thread than for a process. In relation to this, which of the following is **TRUE**?

- (A) On per-thread basis, the OS maintains only CPU register state  
(B) The OS does not maintain a separate stack for each thread  
(C) On per-thread basis, the OS does not maintain virtual memory state  
(D) On per-thread basis, the OS maintains only scheduling and accounting information

[Ans. A]

23. The minimum number of D flip-flops needed to design a mod-258 counter is

- (A) 9 (C) 512  
(B) 8 (D) 258

[Ans. A]

Mod 258 counter has 258 different states. So minimum number of bits required to represent each state is ceiling of  $\log_2 258$  which is 9, each of which requires one flip-flop.

24. The simplified SOP (Sum of Product) form of the Boolean expression

$(P + \bar{Q} + \bar{R}). (P + \bar{Q} + R). (P + Q + \bar{R})$  is

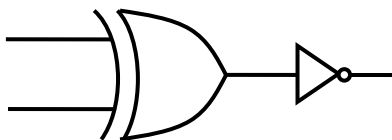
- (A)  $(\bar{P}. Q + \bar{R})$  (C)  $(\bar{P}. Q + R)$   
(B)  $(P + \bar{Q}. \bar{R})$  (D)  $(P. Q + R)$

[Ans. B]

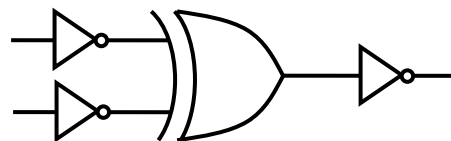
$P = 1$  forces the expression to be 1. This is true only for option B. (Or you can simplify using laws of Boolean algebra and obtain the answer).

25. Which one of the following circuits is **NOT** equivalent to a 2-input XNOR (exclusive NOR) gate?

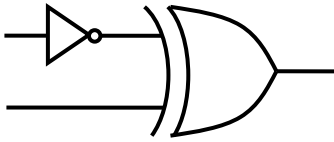
(A)



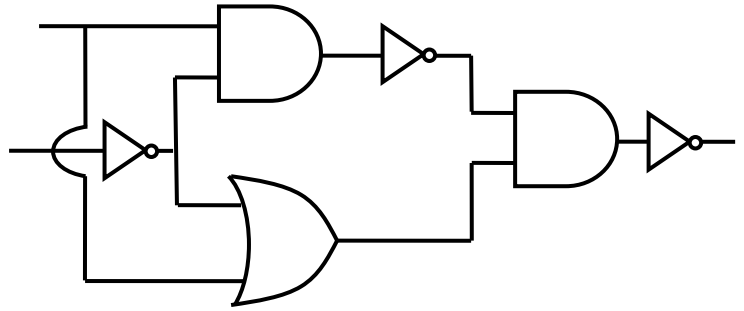
(B)



(C)



(D)



[Ans. D]

Option A is clearly XNOR  $(a \oplus b)'$

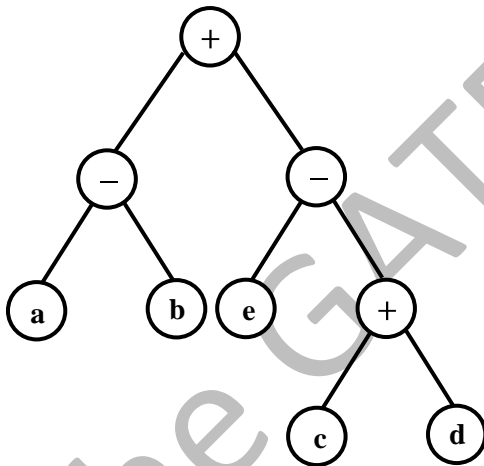
Option B is  $(a' \oplus b')' = ab + a'b' = (a \oplus b)'$  which is XNOR

Option C is  $a' \oplus b = ab + a'b' = (a \oplus b)'$  which is XNOR

Option D is  $((ab)')(a + b'))' = (a \oplus b)$  which is XOR

**Q.26 - Q.55 Carry Two Marks each.**

26. Consider evaluating the following expression tree on a machine with load-store architecture in which memory can be accessed only through load and store instructions. The variables a, b, c, d and e are initially stored in memory. The binary operators used in this expression tree can be evaluated by the machine only when the operands are in registers. The instructions produce result only in a register. If no intermediate results can be stored in memory, what is the minimum number of registers needed to evaluate this expression?



(A) 2

(B) 9

(C) 5

(D) 3

[Ans. D]

At each node in the tree, we can choose to evaluate either the left child or the right child. To minimize the number of registers, we should always choose the one which requires more number of registers, evaluate it and store the result in one of the registers. If both left and right subexpression, required same number of registers, we can evaluate any of them first. The minimum number of registers required in this case is called as **Ershov number** of the expression, and it is evaluated as follows.

Assign 1 to every leaf.

For every internal node, if both its children have same value x, assign  $x + 1$  to the internal node. Otherwise, assign the maximum of the values of its children.

Following this procedure, we obtain 3 as the Ershov number for the given tree.

27. Consider the following table of arrival time and burst time for three processes P0, P1 and P2.

Process	Arrival Time	Burst Time
P0	0 ms	9 ms
P1	1 ms	4 ms
P2	2 ms	9 ms

The pre-emptive shortest job first scheduling algorithm is used. Scheduling is carried out only at arrival or completion of processes. What is the average waiting time for the three processes?

- (A) 5.0 ms (C) 6.33 ms  
(B) 4.33 ms (D) 7.33 ms

[Ans. A]

28. A deck of 5 cards (each carrying a distinct number from 1 to 5) is shuffled thoroughly. Two cards are then removed one at a time from the deck. What is the probability that the two cards are selected with the number on the first card being one higher than the number on the second card?

- (A)  $\frac{1}{5}$  (C)  $\frac{1}{4}$   
(B)  $\frac{4}{25}$  (D)  $\frac{2}{5}$

[Ans. A]

Four (12, 23, 34, 45) out of 20 outcomes are favorable.

29. Consider a finite sequence of random values  $X = [x_1, x_2, \dots, x_n]$ . Let  $\mu_x$  be the mean and  $\sigma_x$  be the standard deviation of X. Let another finite sequence Y of equal length be derived from this as  $y_i = a * x_i + b$ , where a and b are positive constants. Let  $\mu_y$  be the mean and  $\sigma_y$  be the standard deviation of this sequence. Which one of the following statements is **INCORRECT**?

- (A) Index position of mode of X in X is the same as the index position of mode of Y in Y.  
(B) Index position of median of X in X is the same as the index position of median of Y in Y.  
(C)  $\mu_y = a\mu_x + b$   
(D)  $\sigma_y = a\sigma_x + b$

[Ans. D]

$\sigma_y = a\sigma_x$  is the correct expression.

30. Consider a database table T containing two columns X and Y each of type integer. After the creation of the table, one record ( $X = 1, Y = 1$ ) is inserted in the table.

Let MX and MY denote the respective maximum values of X and Y among all records in the table at any point in time. Using MX and MY, new records are inserted in the table 128 times with X and Y values being  $MX + 1, 2 * MY + 1$  respectively. It may be noted that each time after the insertion, values of MX and MY change.

What will be the output of the following SQL query after the steps mentioned above are carried out?

SELECT Y FROM T WHERE X=7;

- (A) 127 (C) 129  
(B) 255 (D) 257

[Ans. A]

The entries inserted in order are

- 1 1  
2 3  
3 7  
4 15  
5 31  
6 63  
7 127

One can also solve the recursion and find out that  $y = 2^x - 1$

31. Given  $i = \sqrt{-1}$ , what will be the evaluation of the definite integral  $\int_0^{\pi/2} \frac{\cos x + i \sin x}{\cos x - i \sin x} dx$ ?
- (A) 0 (C) -i  
(B) 2 (D) i

[Ans. D]

32. Which one of the following options is **CORRECT** given three positive integers  $x, y$  and  $z$ , and a predicate  $P(x) = \neg(x = 1) \wedge \forall y(\exists z(x = y * z) \Rightarrow (y = x) \vee (y = 1))$
- (A)  $P(x)$  being true means that  $x$  is a prime number  
(B)  $P(x)$  being true means that  $x$  is a number other than 1  
(C)  $P(x)$  is always true irrespective of the value of  $x$   
(D)  $P(x)$  being true means that  $x$  has exactly two factors other than 1 and  $x$

[Ans. A]

The given statement reads,

$P(x)$  is true whenever  $x$  is not 1 and for every  $y$ , if there is a  $z$  such that  $x = y * z$ , then either  $x = y$  or  $y = 1$ .  
In other words,  $P(x)$  is true whenever  $x$  is prime.

33. We are given a set of  $n$  distinct elements and an unlabeled binary tree with  $n$  nodes. In how many ways can we populate the tree with the given set so that it becomes a binary search tree?
- (A) 0 (C)  $n!$   
(B) 1 (D)  $\frac{1}{n+1} \cdot {}^{2n}C_n$

[Ans. B]

Given a binary tree, the **inorder** traversal outputs exactly one permutation of its nodes. Since the values are distinct, the values must be inserted in sorted order in these nodes in order. SO, there is only one way.

34. On a non-pipelined sequential processor, a program segment, which is a part of the interrupt service routine, is given to transfer 500 bytes from an I/O device to memory.

Initialize the address register

Initialize the count to 500

Loop: Load a byte from device

Store in memory at address given by address register

Increment the address register

Decrement the count

If count  $\neq 0$  go to LOOP

Assume that each statement in this program is equivalent to a machine instruction which takes one clock cycle to execute if it is a non-load/store instruction. The load-store instructions take two clock cycles to execute.

The designer of the system also has an alternate approach of using the DMA controller to implement the same transfer. The DMA controller requires 20 clock cycles for initialization and other overheads. Each DMA transfer cycle takes two clock cycles to transfer one byte of data from the device to the memory.

What is the approximate speedup when the DMA controller based design is used in place of the interrupt driven program based input-output?

- (A) 3.4 (C) 5.1  
(B) 4.4 (D) 6.7

[Ans. A]

35. Consider the languages  $L_1, L_2$  and  $L_3$  as given below.

$L_1 = \{0^p 1^q | p, q \in \mathbb{N}\}$ ,

$L_2 = \{0^p 1^q | p, q \in \mathbb{N} \text{ and } p = q\}$  and

$L_3 = \{0^p 1^q 0^r | p, q, r \in \mathbb{N} \text{ and } p = q = r\}$ . Which of the following statements is **NOT TRUE**?

- (A) Push Down Automata (PDA) can be used to recognize L1 and L2
- (B) L1 is a regular language
- (C) All the three languages are context free
- (D) Turing machines can be used to recognize all the languages

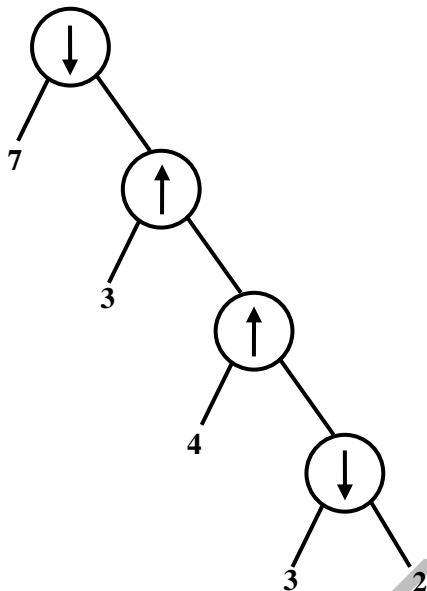
[Ans. C]

L3 is not context free. (It fails to satisfy pumping lemma for context free languages.)

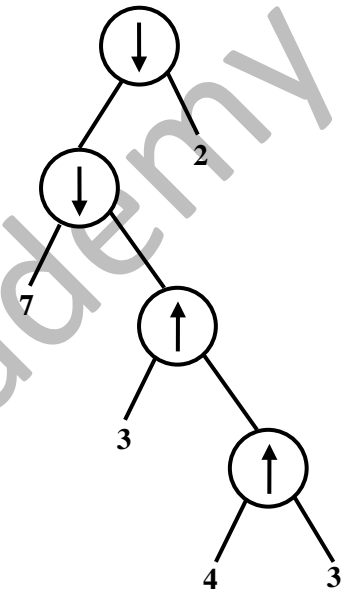
One can also infer the answer by observing that options A, B and D are true.

36. Consider the two binary operators ‘↑’ and ‘↓’ with the precedence of operator ↓ being lower than that of the operator ↑. Operator ↑ is right associative while operator ↓ is left associative. Which one of the following represents the parse tree for expression (7↓3↑4↑3↓2)?

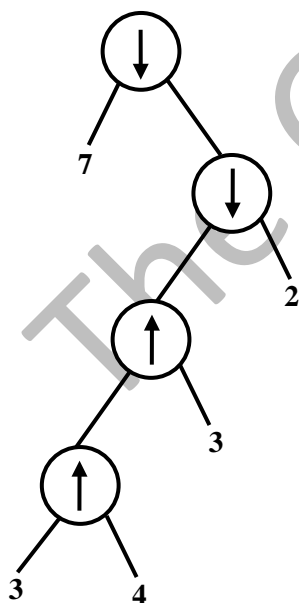
(A)



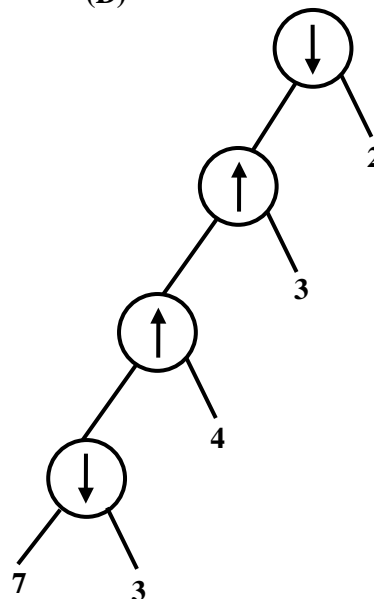
(B)



(C)



(D)



[Ans. B]

When we apply the associativity and precedence rules, we see that  $7\downarrow 3\uparrow 4\uparrow 3\downarrow 2$  is equivalent to the expression

$$(7\downarrow(3\uparrow(4\uparrow 3)))\downarrow 2$$

Clearly, the expression tree is given in option B.

37. Database table by name Loan\_Records is given below.

Borrower	Bank_Manager	Loan_Amount
Ramesh	Sunderajan	10000.00
Suresh	Ramgopal	5000.00
Mahesh	Sunderajan	7000.00

What is the output of the following SQL query?

```
SELECT count(*)
FROM(
    (SELECT Borrower, Bank_Manager FROM Loan_Records) AS S
    NATURAL JOIN
    (SELECT Bank_Manager, Loan_Amount FROM Loan_Records) AS T
);
```

- (A) 3
- (B) 9
- (C) 5
- (D) 6

[Ans. C]

The SQL outputs the number of rows in  
((Ramesh, Sunderajan), (Suresh, Ramgopal), (Mahesh, Sunderajan))

NATURAL JOIN  
((Sunderajan, 10000), (Ramgopal, 5000), (Sunderajan, 7000))

That is,  
Ramesh, Sunderajan, 10000  
Ramesh, Sunderajan, 7000  
Suresh, Ramgopal, 5000  
Mahesh, Sunderajan, 10000  
Mahesh, Sunderajan, 7000  
So, it has 5 rows.

38. The following is the comment written for a C function.

```
/* This function computes the roots of a quadratic equation  $a.x^2 + b.x + c = 0$ . The function stores two real roots in *root1 and *root2 and returns the status of validity of roots. It handles for different kinds of cases.
```

- (i) When coefficient a is zero irrespective of discriminant
- (ii) When discriminant is positive
- (iii) When discriminant is zero
- (iv) When discriminant is negative

Only in case (ii) and (iii), the stored roots are valid. Otherwise 0 is stored in the roots. The function returns 0 when the roots are valid and -1 otherwise.

The function also ensures root1 >= root2.

```
int get_QuadRoots (float a, float b, float c,
    float *root1, float *root2);
```

\*/

A software test engineer is assigned the job of doing black box testing. He comes up with the following test cases, many of which are redundant.

Test Case	Input Set			Expected output set		
	a	b	c	root1	root2	Return Value
T1	0.0	0.0	7.0	0.0	0.0	-1
T2	0.0	1.0	3.0	0.0	0.0	-1
T3	1.0	2.0	1.0	-1.0	-1.0	0
T4	4.0	-12.0	9.0	1.5	1.5	0
T5	1.0	-2.0	-3.0	3.0	-1.0	0
T6	1.0	1.0	4.0	0.0	0.0	-1

Which one of the following options provide the set of non-redundant tests using equivalence class partitioning approach from input perspective for black box testing?

- (A) T1, T2, T3, T6 (C) T2, T4, T5, T6  
(B) T1, T3, T4, T5 (D) T2, T3, T4, T5

[Ans. C]

One can verify the expected output is correct for all test cases.

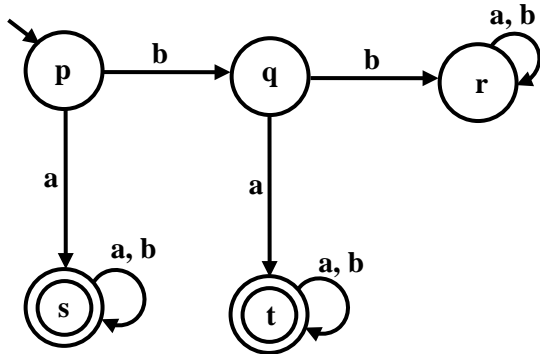
T1 and T2 have  $a = 0$ . (One is redundant)

T3 and T4 have coefficient,  $b^2 - 4ac = 0$  (One is redundant)

T5 has  $b^2 - 4ac > 0$  (It must be present in test plan)

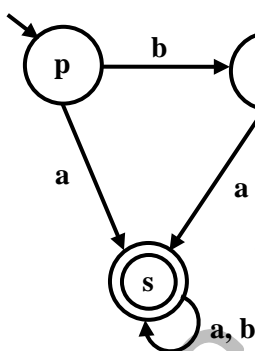
T6 has  $b^2 - 4ac < 0$  (It must be present in test plan)

39. A deterministic finite automata (DFA) D with alphabet  $\Sigma\{a, b\}$  is given below.

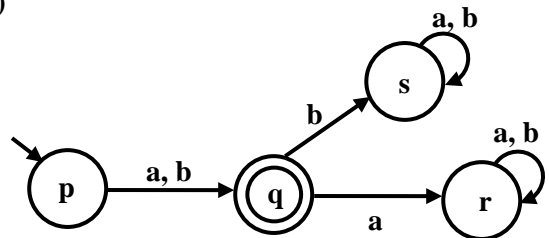


Which of the following finite state machines is a valid minimal DFA which accepts the same language as D?

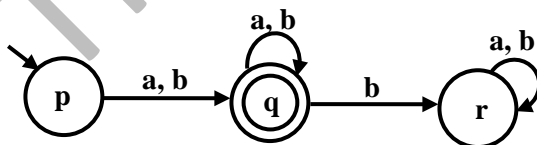
(A)



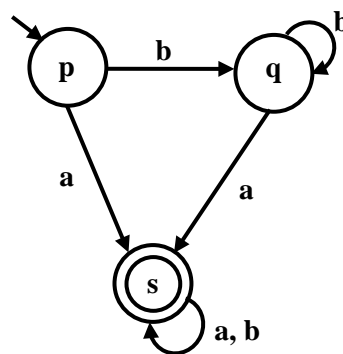
(B)



(C)



(D)



[Ans. A]

One can see that options B, C and D are not equivalent to the given DFA. Options B and C accept input b which is not accepted by input DFA. Option D accepts bba which is not accepted by input DFA.

40. An application loads 100 libraries at startup. Loading each library requires exactly one disk access. The seek time of the disk to a random location is given as 10 ms. Rotational speed of disk is 6000 rpm. If all 100 libraries are loaded from random locations on the disk, how long does it take to load all libraries? (The time to transfer data from the disk block once the head has been positioned at the start of the block may be neglected.)
- (A) 0.50 s (C) 1.25 s  
(B) 1.50 s (D) 1.00 s

[Ans. B]

41. An 8KB direct-mapped write-back cache is organized as multiple blocks, each of size 32-bytes. The processor generates 32-bit addresses. The cache controller maintains the tag information for each cache block comprising of the following.

- 1 Valid bit
- 1 Modified bit

As many bits as the minimum needed to identify the memory block mapped in the cache.

What is the total size of memory needed at the cache controller to store meta-data (tags) for the cache?

- (A) 4864 bits (C) 6656 bits  
(B) 6144 bits (D) 5376 bits

[Ans. A]

42. Definition of a language L with alphabet {a} is given as following

$$L = \{a^{nk} | k > 0, \text{ and } n \text{ is a positive integer constant}\}$$

What is the minimum number of states needed in a DFA to recognize L?

- (A) k + 1 (C)  $2^{n+1}$   
(B) n + 1 (D)  $2^{k+1}$

[Ans. B]

The following DFA that accepts a sequence of a's whose length is a positive multiple of a constant n. It has n + 1 states  $q_0$  through  $q_n$ .

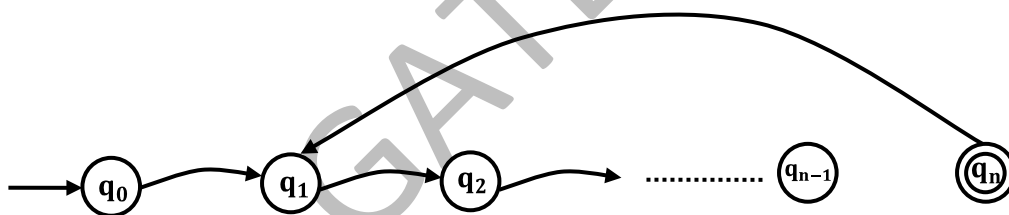
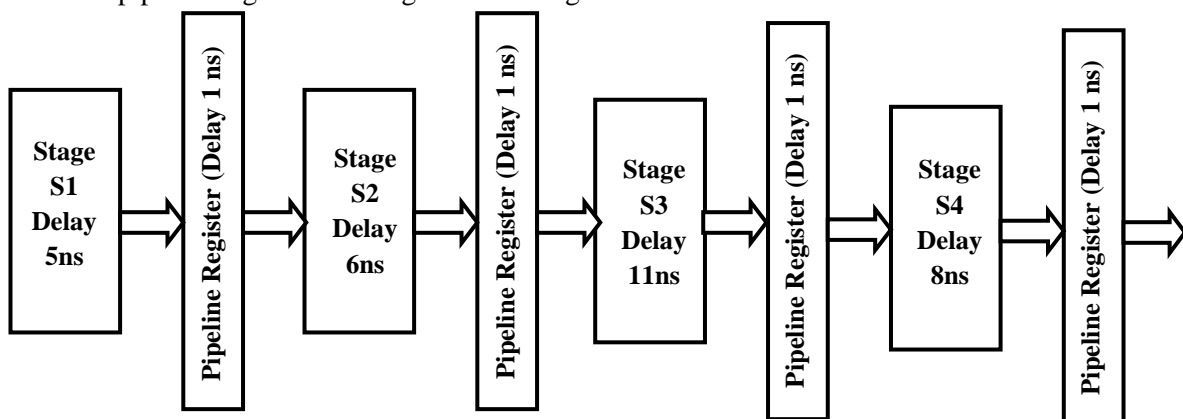


Figure 2: DFA

43. Consider an instruction pipeline with four stages (S1, S2, S3 and S4) each with combinational circuit only. The pipeline registers are required between each stage and at the end of the last stage. Delays for the stages and for the pipeline registers are as given in the figure.



What is the approximate speed up of the pipeline in steady state under ideal conditions when compared to the corresponding non-pipeline implementation?

- (A) 4.0 (C) 1.1  
(B) 2.5 (D) 3.0

[Ans. B]

44. Consider the matrix as given below.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 7 \\ 0 & 0 & 3 \end{bmatrix}$$

Which one of the following options provides the **CORRECT** values of the eigenvalues of the matrix?

- (A) 1, 4, 3 (C) 7, 3, 2  
(B) 3, 7, 3 (D) 1, 2, 3

[Ans. A]

The eigen values of a triangular matrix are given by its diagonal entries. One can also verify the eigen value property, i.e., eigen value  $\lambda$  of a matrix  $M$  must satisfy  $|M - \lambda I| = 0$ .

45. Consider a relational table  $r$  with sufficient number of records, having attributes  $A_1, A_2, \dots, A_n$  and let  $1 \leq p \leq n$ . Two queries  $Q1$  and  $Q2$  are given below.

$Q1: \pi_{A_1, \dots, A_p}(\sigma_{A_p=c}(r))$  where  $c$  is a constant

$Q2: \pi_{A_1, \dots, A_p}(\sigma_{c_1 \leq A_p \leq c_2}(r))$  where  $c_1$  and  $c_2$  are constants

The database can be configured to do ordered indexing on  $A_p$  or hashing on  $A_p$ . Which of the following statements is **TRUE**?

- (A) Ordered indexing will always outperform hashing for both queries  
(B) Hashing will always outperform ordered indexing for both queries  
(C) Hashing outperform ordered indexing on  $Q1$ , but not on  $Q2$   
(D) Hashing will outperform ordered indexing on  $Q2$ , but not on  $Q1$

[Ans. D]

On query  $Q1$ , all values of  $A_p$  are same, and hence there will be hash conflicts for every access, and this hashing is not of much use. But for query  $Q2$ , hashing (constant access time) outperforms ordered indexing (logarithmic access time).

46. Four matrices  $M_1, M_2, M_3$  and  $M_4$  of dimensions  $p \times q, q \times r, r \times s$  and  $s \times t$  respectively can be multiplied in several ways with different number of total scalar multiplications. For example when multiplied as  $((M_1 \times M_2) \times (M_3 \times M_4))$ , the total number of scalar multiplications is  $pqr + rst + prt$ . When multiplied as  $((M_1 \times M_2) \times M_3) \times M_4$ , the total number of scalar multiplications is  $pqr + prs + pst$ .

If  $p = 10, q = 100, r = 20, s = 5$ , and  $t = 80$ , then the minimum number of scalar multiplications needed is

- (A) 248000 (C) 19000  
(B) 44000 (D) 25000

[Ans. C]

By inspection, one can find that the optimum order is  $(M_1(M_2M_3))M_4$ .

The number of multiplications is

$$100 \times 20 \times 5 + 10 \times 100 \times 5 + 10 \times 5 \times 80 = 19000$$

47. Which of the given options provides the increasing order of asymptotic complexity of functions  $f_1, f_2, f_3$  and  $f_4$ ?

$$f_1(n) = 2^n \quad f_2(n) = n^{3/2} \quad f_3(n) = n \log_2 n \quad f_4(n) = n^{\log_2 n}$$

- (A)  $f_3, f_2, f_4, f_1$  (C)  $f_2, f_3, f_1, f_4$   
(B)  $f_3, f_2, f_1, f_4$  (D)  $f_2, f_3, f_4, f_1$

[Ans. A]

$$n \log n \leq n^{3/2} \leq n^{\log n} \leq 2^n$$

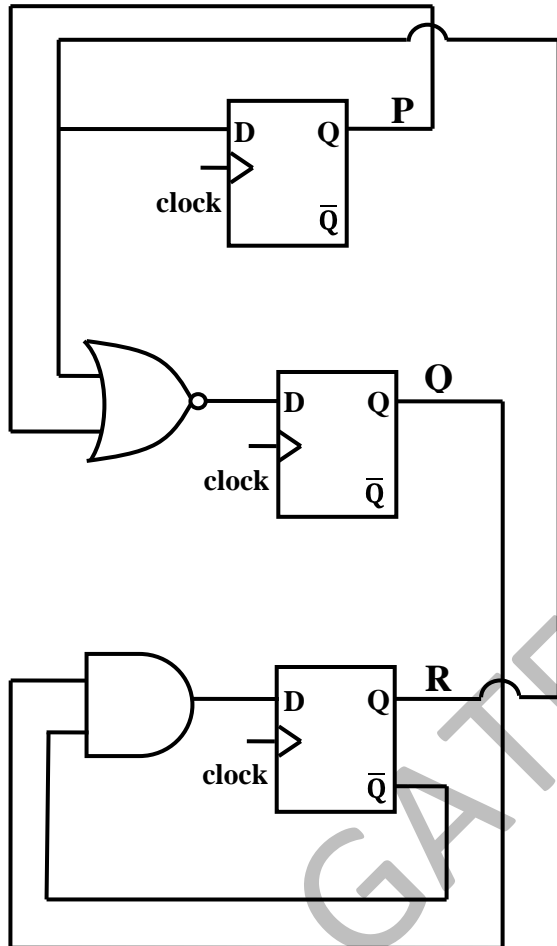
This is clear if we take logarithm of the expressions

$$\log n + \log \log n \leq (3/2) \log n = (\log n)^2 \leq n$$

**Common Data Questions**

**Common Data for Questions 48 and 49:**

Consider the following circuit involving three D-type flip-flops used in a certain type of counter configuration.



48. If at some instance prior to the occurrence of the clock edge, P, Q and R have a value 0, 1 and 0 respectively, what shall be the value of PQR after the clock edge?

- (A) 000
- (B) 001
- (C) 010
- (D) 011

[Ans. D]

$$P' = R$$

$$Q' = (P + R)'$$

$$R' = QR'$$

If (P, Q, R) = (0, 1, 0), the next state (P', Q', R') = (0, 1, 1)

49. If all the flip-flops were reset to 0 at power on, what is the total number of distinct outputs (states) represented by PQR generated by the counter?

- (A) 3
- (B) 4
- (C) 5
- (D) 6

[Ans. B]

There are four distinct states, 000 → 010 → 011 → 100 (→ 000)

**Common Data for Questions 50 and 51:**

Consider the following recursive C function that takes two arguments.

```
unsigned int foo (unsigned int n, unsigned int r) {
    if (n > 0) return ((n%r) + foo (n/r, r));
    else return 0;
}
```

50. What is the return value of the function foo when it is called as foo (345, 10)?

- (A) 345 (C) 5
- (B) 12 (D) 3

**[Ans. B]**

$$\begin{aligned} & \text{foo}(345, 10) \\ &= 5 + \text{foo}(34, 10) \\ &= 5 + 4 + \text{foo}(3, 10) \\ &= 5 + 4 + 3 + \text{foo}(0, 10) \\ &= 12 \end{aligned}$$

51. What is the return value of the function foo when it is called as foo (513, 2)?

- (A) 9 (C) 5
- (B) 8 (D) 12

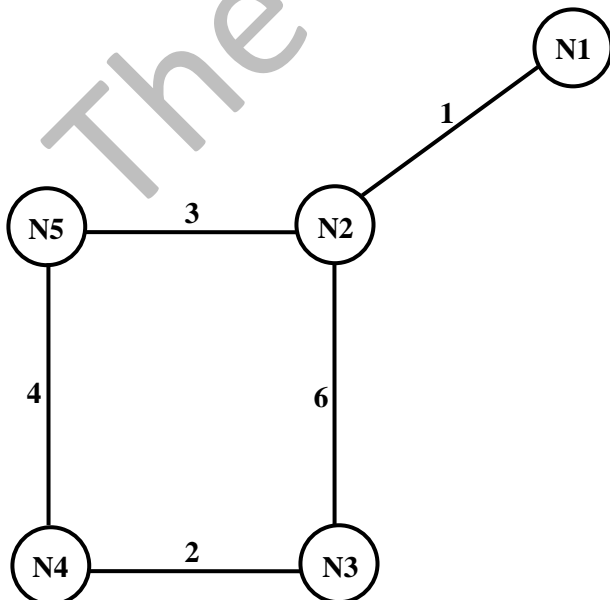
**[Ans. D]**

$$\begin{aligned} & \text{foo}(513, 2) \\ &= 1 + \text{foo}(256, 2) \\ &= 1 + \text{foo}(128, 2) \\ &= 1 + \text{foo}(64, 2) \\ &= 1 + \text{foo}(32, 2) \\ &= 1 + \text{foo}(16, 2) \\ &= 1 + \text{foo}(8, 2) \\ &= 1 + \text{foo}(4, 2) \\ &= 1 + \text{foo}(2, 2) \\ &= 1 + \text{foo}(1, 2) \\ &= 1 + 1 + \text{foo}(0, 2) \\ &= 2 \end{aligned}$$

**Linked Answer Questions:**

**Statement for Linked Questions 52 and 53:**

Consider a network with five nodes, N1 to N5, as shown below.



The network uses a Distance Vector Routing protocol. Once the routes have stabilized, the distance vectors at different nodes are as following.

- N1: (0, 1, 7, 8, 4)
- N2: (1, 0, 6, 7, 3)
- N3: (7, 6, 0, 2, 6)
- N4: (8, 7, 2, 0, 4)
- N5: (4, 3, 6, 4, 0)

Each distance vector is the distance of the best known path at the instance to nodes, N1 to N5, where the distance to itself is 0. Also, all links are symmetric and the cost is identical in both directions. In each round, all nodes exchange their distance vectors with their respective neighbors. Then all nodes update their distance vectors. In between two rounds, any change in cost of a link will cause the two incident nodes to change only that entry in their distance vectors.

52. The cost of link N2-N3 reduces to 2(in both directions). After the next round of updates, what will be the new distance vector at node, N3.
- (A) (3, 2, 0, 2, 5)
  - (B) (3, 2, 0, 2, 6)
  - (C) (7, 2, 0, 2, 5)
  - (D) (7, 2, 0, 2, 6)

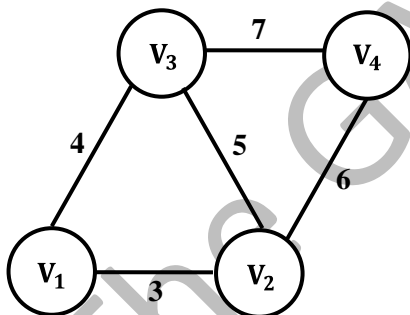
[Ans. A]

53. After the update in the previous question, the link N1-N2 goes down. N2 will reflect this change immediately in its distance vector as cost,  $\infty$ . After the **NEXT ROUND** of update, what will be the cost to N1 in the distance vector of N3?
- (A) 3
  - (B) 9
  - (C) 10
  - (D)  $\infty$

[Ans. C]

**Statement for Linked Answer Questions 54 and 55:**

An undirected graph  $G(V, E)$  contains  $n(n > 2)$  nodes named  $v_1, v_2, \dots, v_n$ . Two nodes  $v_i, v_j$  connected if and only if  $0 < |i - j| \leq 2$ . Each edge  $(v_i, v_j)$  is assigned a weight  $i + j$ . A sample graph with  $n = 4$  is shown below.

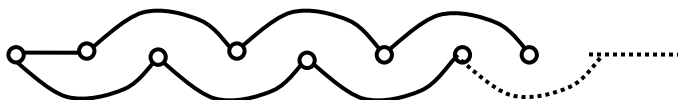


54. What will be the cost of the minimum spanning tree (MST) of such a graph with  $n$  nodes?
- (A)  $\frac{1}{12}(11n^2 - 5n)$
  - (B)  $n^2 - n + 1$
  - (C)  $6n - 11$
  - (D)  $2n + 1$

[Ans. B]

One can test that for  $n = 2$  and  $n = 3$ , the minimum spanning tree weight must be 3 and 7 respectively. We can thus infer that option **B** is the right answer.

The optimum way of constructing the tree is as follows



**Figure 3: Minimum spanning tree**

A new vertex  $v_{n+1}$  added is made adjacent to the  $v_{n-1}$  in the spanning tree. The spanning tree weight increases by  $(n + 1) + (n - 1) = 2n$

One can verify that  $(n^2 - n + 1) + 2n = (n + 1)^2 - (n + 1) - 1$

55. The length of the path from  $v_5$  to  $v_6$  in the MST of previous questions with  $n = 10$  is

- (A) 11 (C) 31  
(B) 25 (D) 41

[Ans. C]

Let  $e_{ij}$  denote the edge from  $v_i$  to  $v_j$  in spanning tree.

The path from  $v_5$  to  $v_6$  is given by  $e_{53}e_{31}e_{12}e_{24}e_{46}$  and its weight is  $5 + 3 + 3 + 1 + 1 + 2 + 2 + 4 + 4 + 6 = 31$

### General Aptitude (GA) Questions

**Q. 56 – Q. 60 carry one mark each.**

56. Choose the most appropriate word (s) from the options given below to complete the following sentence.

**I Contemplated \_\_\_\_\_ Singapore for my vacation but decided against it.**

- (A) to visit (C) visiting  
(B) having to visit (D) for a visit

[Ans. C]

Contemplate is a transitive verb and hence is followed by a gerund. Hence the correct usage of contemplate is verb+ ing form.

57. Choose the most appropriate word from the options given below to complete the following sentence.

**If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or \_\_\_\_\_.**

- (A) hyperbolic (C) argumentative  
(B) restrained (D) indifferent

[Ans. B]

The tone of the sentence clearly indicates a word that is similar to understated is needed for the blank. Alternatively, the word should be antonym of strong (fail to make strong impression). Therefore, the best choice is restrained which means controlled/reserved/timid.

58. Choose the word from the options given below that is most nearly opposite in meaning to the given word:

**Amalgamate**

- (A) merge (C) collect  
(B) split (D) separate

[Ans. B]

Amalgamate means combine or unite to form one organization or structure. SO the best option here is split. Separate on the other hand, although a close synonym, it is too general to be the best antonym in the given question while Merge is the synonym; Collect is not related.

59. Which of the following options is the closest in the meaning to the word below:

**Inexplicable**

- (A) incomprehensible (C) inextricable  
(B) indelible (D) infallible

[Ans. A]

Inexplicable means not explicable; that cannot be explained, understood, or accounted for. So the best synonym here is incomprehensible.

60. If  $\log(P) = (1/2) \log(Q) = (1/3) \log(R)$ , then which of the following options is **TRUE**?

- (A)  $P^2 = Q^3 R^2$  (C)  $Q^2 = R^3 P$   
(B)  $Q^2 = PR$  (D)  $R = P^2 Q^2$

[Ans. B]

$$\log P = \frac{1}{2} \log Q = \frac{1}{3} \log(R) = k$$

$$\therefore P = b^k, Q = b^{2k}, R = b^{3k}$$

$$\text{Now, } Q^2 = b^{4k} = b^{3k} b^k = PR$$

**Q. 61 to Q. 65 carry two marks each**

61. The variable cost (V) of manufacturing a product varies according to the equation  $V = 4q$ , where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation  $F = 100/q$ . How many units should be produced to minimize the total cost (V + F)?

- (A) 5 (C) 7  
(B) 4 (D) 6

[Ans. A]

Checking with all options in formula:  $(4q + 100/q)$  i.e. (V + F). Option A gives the minimum cost.

62. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4<sup>th</sup> day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10<sup>th</sup> day. What is the minimum number of trucks required so that there will be no pending order at the end of the 5<sup>th</sup> day?

- (A) 4 (C) 6  
(B) 5 (D) 7

[Ans. C]

Let each truck carry 100 units.

$$2800 = 4n + e \quad n = \text{normal}$$

$$3000 = 10n + e \quad e = \text{excess/pending}$$

$$\therefore n = \frac{100}{3}, e = \frac{8000}{3}$$

$$5 \text{ days} \Rightarrow 500x = \frac{5 \cdot 100}{3} + \frac{8000}{3}$$

$$\Rightarrow 500x = \frac{8500}{3} \Rightarrow x > 5$$

Minimum possible = 6

63. A container originally contains 10 litres of pure spirit. From this container 1 litre of spirit is replaced with 1 litre of water. Subsequently, 1 litre of the mixture is again replaced with 1 litre of water and this process is repeated one more time. How much spirit is now left in the container?

- (A) 7.58 litres (C) 7 litres  
(B) 7.84 litres (D) 7.29 litres

[Ans. D]

$$10 \left( \frac{10-1}{10} \right)^3 = 10 \left( \frac{9}{10} \right)^3 = \frac{729}{1000}$$

$$\therefore \frac{729}{1000} \times 1 = 7.29 \text{ litres}$$

64. **Few school curricula include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.**

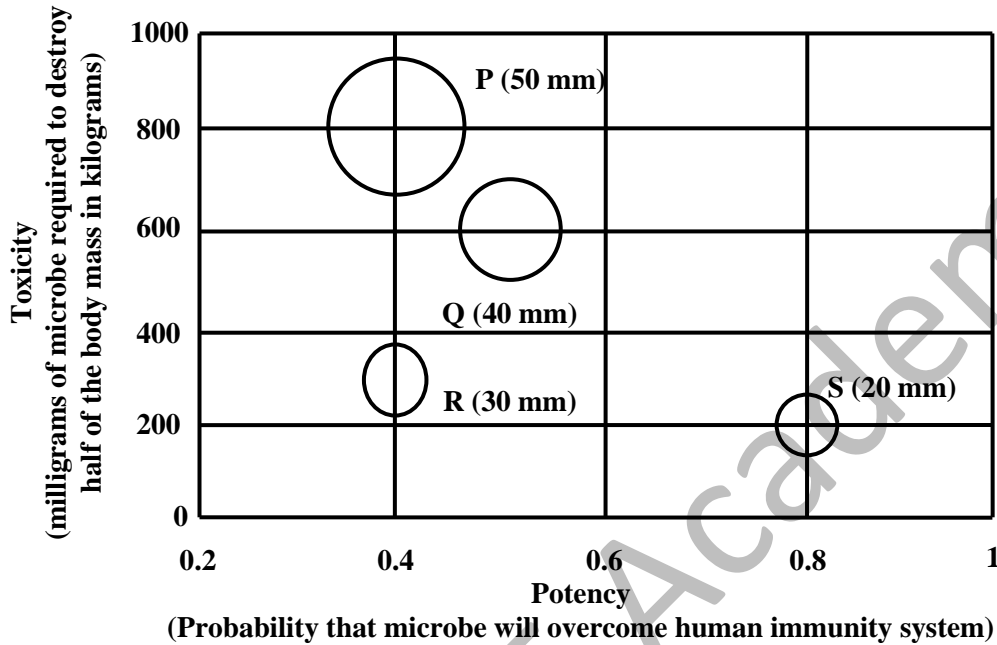
Based on the above passage which topic would not be included in a unit on bereavement?

- (A) how to write a letter of condolence  
(B) what emotional stages are passed through in the healing process  
(C) what the leading causes of death are  
(D) how to give support to a grieving friend

[Ans. C]

The given passage clearly deals with how to deal with bereavement and grief and so after the tragedy occurs and not about precautions. Therefore, irrespective of the causes of death, a school student rarely gets into details of causes – which is beyond the scope of the context. Rest all are important in dealing with grief.

65. P, Q, R and S are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system with 24 hours of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure below:



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

- (A) P (C) R  
(B) Q (D) S

[Ans. D]

By observation of the table, we can say S

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8